Sheffield Hallam University

LAPLACE TRANSFORM TABLES

DEFINITION

The Laplace transform $\bar{f}(s)$ of a function f(t) is defined by:

$$\bar{f}(s) = \int_0^\infty e^{-st} \quad f(t)dt$$

TRANSFORMS OF STANDARD FUNCTIONS

f(t)	$\bar{f}(s)$
1	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
$\frac{1}{T}e^{-\frac{t}{T}}$	$\frac{1}{1+sT}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
te^{-at}	$\frac{1}{(s+a)^2}$
$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$e^{-at}t^n$	$\frac{n!}{(s+a)^{n+1}}$
$\sin wt$	$\frac{w}{s^2+w^2}$
$\cos wt$	$\frac{s}{s^2+w^2}$

f(t)	$\bar{f}(s)$
$e^{-at}\sin wt$	$\dfrac{w}{(s+a)^2+w^2}$
$e^{-at}\cos wt$	$\dfrac{s+a}{(s+a)^2+w^2}$
$1-\cos wt$	$\dfrac{w^2}{s(s^2+w^2)}$
$\dfrac{1}{2w^3}(\sin wt - wt\cos wt)$	$\dfrac{1}{(s^2+w^2)^2}$
$\dfrac{t}{2w}\sin wt$	$\dfrac{s}{(s^2+w^2)^2}$
$e^{-at}\left(\cos wt - \dfrac{a}{w}\sin wt\right)$	$\dfrac{s}{(s+a)^2+w^2}$
$\sin(wt+f)$	$\dfrac{s\sin f + w\cos f}{s^2+w^2}$
$e^{-at}+\dfrac{a}{w}\sin wt-\cos wt$	$\dfrac{a^2+w^2}{(s+a)(s^2+w^2)}$
$\sin^2 wt$	$\dfrac{2w^2}{s(s^2+4w^2)}$
$\cos^2 wt$	$\dfrac{s^2+2w^2}{s(s^2+4w^2)}$
$\sinh bt$	$\dfrac{b}{s^2-b^2}$
$\cosh bt$	$\dfrac{s}{s^2-b^2}$

f(t)	$\bar{f}(s)$
$e^{-at}\sinh bt$	$\dfrac{b}{(s+a)^2-b^2}$
$e^{-at}\cosh bt$	$\dfrac{s+a}{(s+a)^2-b^2}$
$t\sinh bt$	$\dfrac{2bs}{\left(s^2-b^2\right)^2}$
$t\cosh bt$	$\dfrac{s^2+b^2}{\left(s^2-b^2\right)^2}$
$\dfrac{1}{2b^3}\left(bt\cosh bt-\sinh bt\right)$	$\dfrac{1}{\left(s^2-b^2\right)^2}$

Transforms of Special Functions

Unit impulse :	$\delta(t)$	1
Unit step :	H(t)	$\dfrac{1}{s}$
Ramp:	tH(t)	$\dfrac{1}{s^2}$
Delayed Unit Impulse:	$\delta(t-T)$	e^{-sT}
Delayed Unit Step:	H(t-T)	$\dfrac{e^{-sT}}{s}$
Rectangular Pulse:	H(t)-H(t-T)	$\dfrac{1-e^{-sT}}{s}$

TRANSFORM THEOREMS

	f(t)	$\bar{f}(s)$
Damping:	$e^{-\alpha t}f(t)$	$\bar{f}(s+a)$
Delay:	f(t-T)H(t-T)	$e^{-sT}\bar{f}(s)$
Time scale:	f(kt)	$\frac{1}{k}\bar{f}\left(\frac{s}{k}\right)$
Integral:	$\int_0^t f(t)dt$	$\frac{1}{s}\bar{f}(s)$
Differentiation	$\frac{d}{dt}f(t)$	$s\bar{f}(s)-f(0)$
	$\frac{d^2}{dt^2}f(t)$	$s^2\bar{f}(s)-sf(0)-f'(0)$
	$\frac{d^n}{dt^n}f(t)$	$s^n\bar{f}(s)-s^{n-1}f(0)-s^{n-2}f'(0)-\ldots-f^{n-1}(0)$

Initial Value: $\lim_{t\to 0}\{f(t)\}=\lim_{s\to\infty}\{s\bar{f}(s)\}$

Final Value: $\lim_{t\to\infty}\{f(t)\}=\lim_{s\to 0}\{s\bar{f}(s)\}$

Periodic Functions: If f(t) has period T then: $\bar{f}(s)=\frac{1}{1-e^{-sT}}\int_0^T f(t)e^{-st}dt$

further, if g(t) is defined as the first cycle of f(t), followed by zero, then $\bar{f}(s)=\frac{\bar{g}(s)}{1-e^{-sT}}$

Square Wave:

$$\left.\begin{array}{ll} f(t)=1 & 0<t<\frac{T}{2} \\ f(t)=0 & \frac{T}{2}<t<T \end{array}\right\} \quad \bar{f}(s)=\frac{1}{2s}\left[1+\frac{e^{aT}-e^{-aT}}{e^{aT}+e^{-aT}}\right], \textit{ where } \quad a=\frac{s}{4}$$

Half-Wave Rectified Sine:

$$\left.\begin{array}{ll} f(t)=\sin w\ t & 0<t<\frac{p}{w} \\ f(t)=0 & \frac{p}{w}<t<\frac{2p}{w} \end{array}\right\} \quad \bar{f}(s)=\frac{w}{s^2+w^2}\cdot\frac{1}{1-e^{-sp/w}}$$

Full-Wave Rectified Sine: $f(t)=|\sin w\ t|$ $\quad \bar{f}(s)=\frac{w}{s^2+w^2}\frac{1+e^{-sp/w}}{1-e^{-sp/w}}$

Saw-Tooth Wave: $f(t) = \dfrac{t}{T} \quad 0 < t < T \qquad \bar{f}(s) = \dfrac{1}{s^2 T} - \dfrac{e^{-sT}}{s\left(1 - e^{-sT}\right)}$