1-

Matriz Simétrica: Propiedad

$$A^T = A$$

$$\begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix}^T = \begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix} = A \Rightarrow \textbf{\textit{SI}}$$

$$\begin{bmatrix} 7 & 2 & 0 \\ 3 & 5 & -1 \\ 0 & 5 & -6 \end{bmatrix}^T = \begin{bmatrix} 7 & 3 & 0 \\ 2 & 5 & 5 \\ 0 & -1 & -6 \end{bmatrix} \neq A \Rightarrow \textbf{\textit{NO}}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix}^T = A \Rightarrow \textbf{\textit{SI}}$$

Matriz Singular

$$Det(A) = 0$$

$$Det\left(\begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix}\right) = -2(-3) - 1 * 1 = 5 = A \Rightarrow \textbf{\textit{NO}}$$

$$Det\left(\begin{bmatrix} 7 & 2 & 0 \\ 3 & 5 & -1 \\ 0 & 5 & -6 \end{bmatrix}\right) = 7(5*(-6) - 5*(-1)) - 2*(3*(-6) - 0*(-1)) + 0*(3*5 - 0*5) = -139 \ \textbf{\textit{NO}}$$

$$Det\left(\begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix}\right) 2*(4*2 - 2*2) + 1(-1*2) = 6 \ \textbf{\textit{NO}}$$

Dominante diagonalmente

$$\forall a_{ii} \Rightarrow |a_{ii}| \geq \sum_{\substack{ij \\ j\neq i}}^{n} |a_{ij}|$$

$$\textbf{\textit{SI}} \begin{cases} 2 > 1 \\ 3 > 1 \end{cases} \quad \textbf{\textit{SI}} \begin{cases} 7 > 2 \\ 5 > 3 + 1 \\ 6 > 5 \end{cases} \quad \textbf{\textit{SI}} \begin{cases} 2 > 1 \\ 4 > 1 + 2 \\ 2 \geq 2 \end{cases}$$

2-

Autovalores

$$Sea\ \lambda\ un\ autovalor \Longrightarrow |A - \lambda I| = 0$$

$$\left\|\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix}\right\| = (2-\lambda)^2 - 1 = \lambda^2 - 4\lambda + 3 \Rightarrow \boldsymbol{A = \{1; 3\}}$$

$$\begin{bmatrix} 2-\lambda & 1 & 0 \\ 1 & 2-\lambda & 0 \\ 0 & 0 & -5-\lambda \end{bmatrix} = -x^3 - x^2 + 17x - 15 \Rightarrow \boldsymbol{B = \{1, 3, -5\}}$$

Radio Espectral

$$\rho(A) = Max\,|\lambda_i|$$

$$\boldsymbol{\rho(A) = 3}$$
$$\boldsymbol{\rho(B) = 5}$$

Norma 2

$$||A||_2 = \sqrt{\rho(A^*A)}\ Donde\ A^*\ es\ la\ Transpuesta\ Conjugada$$

$$\begin{cases} \sqrt{\rho\left(\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} * \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\right)} = \sqrt{\rho\left(\begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}\right)} \Rightarrow \sqrt{9} = \boldsymbol{3} \\ \left\|\begin{bmatrix} 5-\lambda & -4 \\ -4 & 5-\lambda \end{bmatrix}\right\| = x^2 - 10*x + 9 \end{cases}$$

$$\begin{cases} \sqrt{\rho\left(\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & -5 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & -5 \end{bmatrix}\right)} = \sqrt{\rho\left(\begin{bmatrix} 5 & 4 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 25 \end{bmatrix}\right)} \Rightarrow \sqrt{25} = \boldsymbol{5} \\ \left\|\begin{bmatrix} 5-\lambda & -4 \\ -4 & 5-\lambda \end{bmatrix}\right\| = x^2 - 10*x + 9 \end{cases}$$

3-

Norma Infinito

$$||A||_\infty = \max_{1<i<n} \sum_{j=1}^{n} |a_{ij}|$$

"Es la suma máxima en fila, que resulta de sumar los módulos de los coeficientes fila"

Norma 1

$$||A||_1 = \max_{1<j<n} \sum_{i=1}^{n} |a_{ij}|$$

"Es la suma máxima encolumna, que resulta de sumar los módulos de los coeficientes de la misma"

$$||A||_\infty = Max\{17; 10; 16\} = \boldsymbol{17}$$

$$||A||_1 = Max\{16; 11; 16\} = \boldsymbol{16}$$

4-

$$\begin{cases} |2k-3| \geq 4+1 \\ 5 > 4 \\ |10-k| \geq 6 \end{cases} \Rightarrow \begin{array}{c} \left(\left[si\ k \geq \frac{3}{2} \wedge 2k-3 \geq 5\right] \vee \left[si\ k \leq \frac{3}{2} \wedge 2k-3 \leq -5\right]\right) \\ \wedge \\ ([si\ k \leq 10 \wedge 10-k \geq 6] \vee [si\ k > 10 \wedge 10-k < -6]) \end{array} \Rightarrow \begin{array}{c} ([4;\infty) \cup (-\infty;-1]) \\ \cap \\ [(-\infty;4] \cup [16,\infty)] \end{array}$$

Para que sea estricto debemos quitar los extremos:

$$\boldsymbol{Sol}: \begin{cases} \boldsymbol{k \in (-\infty,-1] \cup \{4\} \cup [16,+\infty)\ DD} \\ \boldsymbol{k \in (-\infty;-1) \cup (16;+\infty)\ EDD} \end{cases}$$

$$\begin{cases} |3k-3| \geq 4+1 \\ |5+k^2| \geq 4 \\ |10-k| \geq 11 \end{cases} \Rightarrow \begin{array}{c} ([si\ k \geq 1 \wedge 3k-3 \geq 5] \vee [si\ k \leq 1 \wedge 3k-3 \leq -5]) \\ \wedge \\ (\forall k) \\ \wedge \\ ([k \leq 10 \wedge k \leq 1] \vee [k \geq 10 \wedge k \geq 21) \end{array} \Rightarrow \begin{array}{c} \left(\left[\frac{8}{3};\infty\right) \cup \left(-\infty;-\frac{2}{3}\right]\right) \\ \cap \\ [(-\infty;-1] \cup [21,\infty)] \end{array}$$

Para que sea estricto debemos quitar los extremos:

$$\boldsymbol{Sol}: \begin{cases} \boldsymbol{k \in (-\infty,-1] \cup [21,+\infty)\ DD} \\ \boldsymbol{k \in (-\infty;-1) \cup (21;+\infty)\ EDD} \end{cases}$$

5-

Para aplicar el método de Jacobi despejamos de cada ecuación del sistema una incógnita distinta y luego iteramos sobre el resultado de esa matriz iniciando en el valor inicial propuesto:

$$\begin{cases} x_1 = \frac{8}{7} + \frac{1}{7}x_2 - \frac{4}{7}x_3 \\ x_2 = \frac{1}{2} + \frac{3}{8}x_1 + \frac{1}{4}x_3 \\ x_3 = -\frac{1}{2} + \frac{2}{3}x_1 + \frac{1}{6}x_2 \end{cases}$$

$$\boldsymbol{X^{(0)} = (0\,;0;0)}$$

$$\boldsymbol{X^{(1)} = \left(\frac{8}{7};\frac{1}{2};-\frac{1}{2}\right)} \Rightarrow \left\|X^{(1)} - X^{(0)}\right\|_\infty = \frac{15}{7}$$

$$\boldsymbol{X^{(2)} = \left(\frac{3}{2};\frac{45}{56};\frac{29}{84}\right)} \Rightarrow \left\|X^{(2)} - X^{(1)}\right\|_\infty = \left\|\left(\frac{5}{14};\frac{17}{56};\frac{71}{84}\right)\right\|_\infty = \frac{253}{168}$$

$$\boldsymbol{X^{(3)} = \left(\frac{1247}{1176};\frac{193}{168};\frac{71}{112}\right)} \Rightarrow \left\|X^{(3)} - X^{(2)}\right\|_\infty = \left\|\left(-\frac{517}{1176};\frac{29}{84};\frac{97}{336}\right)\right\|_\infty = \frac{2525}{2352}$$

$$\boldsymbol{X^{(4)} = \left(\frac{1111}{1176};\frac{207}{196};\frac{937}{2352}\right)} \Rightarrow \left\|X^{(4)} - X^{(3)}\right\|_\infty = \left\|\left(-\frac{17}{147};-\frac{109}{1176};-\frac{277}{1176}\right)\right\|_\infty = \frac{87}{196}$$

$$\boldsymbol{X^{(5)} = \left(\frac{1097}{1029};\frac{641}{672};\frac{1079}{3528}\right)} \Rightarrow \left\|X^{(5)} - X^{(4)}\right\|_\infty = \left\|\left(\frac{333}{2744};-\frac{481}{4704};-\frac{653}{7056}\right)\right\|_\infty = 0.31757$$

$$\boldsymbol{X^{(6)} = (1.10435901; 0.97624109; 0.36970056)} \Rightarrow \left\|X^{(6)} - X^{(5)}\right\|_\infty = \|(0.038; 0.0223; 0.063)\|_\infty = 0.1233$$

$$\boldsymbol{X^{(7)} = (1.07106269; 1.006555977; 0.39894619)} \Rightarrow \left\|X^{(7)} - X^{(6)}\right\|_\infty = \|(-0.033; 0.03; 0.029)\|_\infty = 0.092$$

$$\boldsymbol{X^{(8)} = (1.05868215; 1.00138506; 0.38544035)} \Rightarrow \left\|X^{(8)} - X^{(7)}\right\|_\infty = \|(-0.012; -0.005; -0.013)\|_\infty = 0.03$$

En la próxima Iteración ya se pasa la precisión. Por lo tanto es $\mathbf{X^{(8)}}$

6-

Para aplicar el método de Gauss-Seidel realizamos la misma operación que para Jacobi, pero al pasar a la otra ecuación del sistema ya reemplazamos el valor de la incógnita hallada:

$$\begin{cases} x_1 = \frac{8}{7} + \frac{1}{7}x_2 - \frac{4}{7}x_3 \\ x_2 = \frac{1}{2} + \frac{3}{8}x_1 + \frac{1}{4}x_3 \\ x_3 = -\frac{1}{2} + \frac{2}{3}x_1 + \frac{1}{6}x_2 \end{cases}$$

$$\boldsymbol{X^{(0)} = (0\,;0;0)}$$

$$\boldsymbol{X^{(1)} = \left(\frac{8}{7};\frac{13}{14};\frac{5}{12}\right)} \Rightarrow \left\|X^{(1)} - X^{(0)}\right\|_\infty = \frac{209}{84}$$

$$\boldsymbol{X^{(2)} = \left(\frac{305}{294};\frac{146}{147};\frac{5}{14}\right)} \Rightarrow \left\|X^{(2)} - X^{(1)}\right\|_\infty = \frac{45}{196}$$

$$\boldsymbol{X^{(3)} = \left(\frac{1112}{1029};\frac{2729}{2744};\frac{2725}{7056}\right)} \Rightarrow \left\|X^{(3)} - X^{(2)}\right\|_\infty = \frac{143}{1942}$$

$$\boldsymbol{X^{(4)} = (1.06424985\,; 0.99564273; 0.375440355)} \Rightarrow \left\|X^{(4)} - X^{(3)}\right\|_\infty = \frac{175}{6189}$$

En la próxima Iteración ya se pasa la precisión. Por lo tanto es $\mathbf{X^{(4)}}$

<u>T y C de Jacobi y Gauss-Seiddel</u>

$$\boldsymbol{T_J = D^{-1}(L+U)\ \ y\ \ C_J = D^{-1}B}$$

$$\boldsymbol{T_{GS} = (D-L)^{-1}U\ \ C_{GS} = (D-L)^{-1}B}$$

$$\boldsymbol{Siendo\ X^{-1} = \frac{1}{|X|}Adj(X)}$$

$$\begin{cases} A = D - L - U = \begin{bmatrix} 7 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -6 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ -3 & 0 & 0 \\ -4 & -1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 & -4 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \\ D^{-1} = \frac{1}{336}\begin{bmatrix} 48 & 0 & 0 \\ 0 & -42 & 0 \\ 0 & 0 & -56 \end{bmatrix}^T = \begin{bmatrix} \frac{1}{7} & 0 & 0 \\ 0 & -\frac{1}{8} & 0 \\ 0 & 0 & -\frac{1}{6} \end{bmatrix} \\ (D-L)^{-1} = \frac{1}{336}\begin{bmatrix} 48 & 18 & 35 \\ 0 & -42 & -7 \\ 0 & 0 & -56 \end{bmatrix}^T = \begin{bmatrix} \frac{1}{7} & 0 & 0 \\ \frac{3}{56} & -\frac{1}{8} & 0 \\ \frac{5}{48} & -\frac{1}{48} & -\frac{1}{6} \end{bmatrix} \end{cases} \Rightarrow$$

$$T_J = \begin{bmatrix} \frac{1}{7} & 0 & 0 \\ 0 & -\frac{1}{8} & 0 \\ 0 & 0 & -\frac{1}{6} \end{bmatrix} \begin{bmatrix} 0 & 1 & -4 \\ -3 & 0 & -2 \\ -4 & -1 & 0 \end{bmatrix} = \boldsymbol{\begin{bmatrix} 0 & \frac{1}{7} & -\frac{4}{7} \\ \frac{3}{8} & 0 & \frac{1}{4} \\ \frac{2}{3} & \frac{1}{6} & 0 \end{bmatrix}}$$

Observen que la Matriz de Jacobi coincide con la Matriz con la que hallamos $X^{(i)}$ sin la columna de términos independientes.

$$T_{GS}=\begin{bmatrix}\frac{1}{7} & 0 & 0\\ \frac{3}{56} & -\frac{1}{8} & 0\\ \frac{5}{48} & -\frac{1}{48} & -\frac{1}{6}\end{bmatrix}\begin{bmatrix}0 & 1 & -4\\ 0 & 0 & -2\\ 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}\mathbf{0} & \frac{\mathbf{1}}{\mathbf{7}} & -\frac{\mathbf{4}}{\mathbf{7}}\\ \mathbf{0} & \frac{\mathbf{3}}{\mathbf{56}} & \frac{\mathbf{1}}{\mathbf{28}}\\ \mathbf{0} & \overset{\mathbf{5}}{-\frac{\mathbf{5}}{\mathbf{48}}} & \mathbf{-3/8}\end{bmatrix}$$

7-

Una de las propiedades indica que si la matriz de Coeficientes asociada es Diagonalmente Dominante entonces converge al valor buscado:

$$\begin{cases}4>1\\3>1\end{cases}\Rightarrow DD\Rightarrow CONVERGE$$

F(4(cifras Decimales),Decimal,-,-)

$$\begin{cases}x=\frac{1}{4}+\frac{1}{4}y\\ y=\frac{2}{3}-\frac{1}{3}x\end{cases}$$

$$\boldsymbol{X^{(0)}=(0\,;0);\ X^{(1)}=\left(\frac{1}{4};\frac{7}{12}\right);\ X^{(2)}=\left(\frac{19}{48};\frac{77}{144}\right);\ X^{(3)}=\left(\frac{221}{576};\frac{931}{1728}\right);X^{(4)}=\left(\frac{2659}{6912};0,5384\right)}$$

$$\boldsymbol{X^{(5)}=(0,3846;0,5385)}$$

$$\boldsymbol{X^{(0)}=(9\,;5);\ X^{(1)}=\left(\frac{3}{2};\frac{1}{6}\right);\ X^{(2)}=\left(\frac{7}{24};\frac{41}{72}\right);\ X^{(3)}=\left(\frac{113}{288};\frac{463}{864}\right);X^{(4)}=\left(\frac{1327}{3456};\frac{5585}{10368}\right)}$$

$$\boldsymbol{X^{(5)}=(0,3847;0,5384)}$$

8-

Una de las propiedades indica que si la matriz de Coeficientes asociada es Diagonalmente Dominante entonces converge al valor buscado:

$$\begin{cases}2>1\\6>3\end{cases}\Rightarrow DD\Rightarrow CONVERGE$$

$$\begin{cases}x=4-\frac{1}{2}y\\ y=\frac{7}{2}-\frac{1}{2}x\end{cases}$$

$$\boldsymbol{X^{(0)}=(1;4);\ X^{(1)}=(2;3);\ X^{(2)}=\left(\frac{5}{2};\frac{5}{2}\right);\ X^{(3)}=\left(\frac{11}{4};\frac{9}{4}\right);X^{(4)}=\left(\frac{23}{8};\frac{17}{8}\right)}$$

$$\boldsymbol{X^{(5)}=\left(\frac{47}{16};\frac{33}{16}\right);\ X^{(6)}=\left(\frac{95}{32};\frac{65}{32}\right)}$$

$$\boldsymbol{X^{(0)}=(1;4);\ X^{(1)}=\left(2;\frac{5}{2}\right);\ X^{(2)}=\left(\frac{11}{4};\frac{17}{8}\right);\ X^{(3)}=\left(\frac{47}{16};\frac{65}{32}\right);X^{(4)}=\left(\frac{191}{64};\frac{257}{128}\right)}$$

$$\boldsymbol{X^{(5)}=\left(\frac{767}{256};\frac{1025}{512}\right);\ X^{(6)}=\left(\frac{3071}{1024};\frac{4097}{2048}\right)}$$

9-

a)

Primero buscamos que la matriz de coeficientes sea DD para asegurar la convergencia, modificando las filas de la misma:

$$\begin{cases}8x_1-5x_3=16\\ x_1+3x_2=0\\ -x_2+6x_3=-40\end{cases}\Rightarrow\begin{cases}x_1=2+\frac{5}{8}x_3\\ x_2=-\frac{1}{3}x_1\\ x_3=-\frac{20}{3}+\frac{1}{6}x_2\end{cases}$$

$$\boldsymbol{X^{(1)}=(0;0;0)}$$

$$\boldsymbol{X^{(1)}=\left(2;-\frac{2}{3};-\frac{61}{9}\right)}\Rightarrow\left\|X^{(1)}-X^{(0)}\right\|_\infty=\frac{85}{9}$$

$$\boldsymbol{X^{(2)}=\left(-\frac{161}{72};\frac{161}{216};-\frac{1773}{271}\right)}\Rightarrow\left\|X^{(2)}-X^{(1)}\right\|_\infty=\left\|\left(\frac{305}{72};-\frac{305}{216};-\frac{574}{2439}\right)\right\|_\infty=\frac{606}{103}$$

$$\boldsymbol{X^{(3)}=\left(-\frac{1713}{820};\frac{571}{820};-\frac{1618}{247}\right)}\Rightarrow\left\|X^{(3)}-X^{(2)}\right\|_\infty=\left\|\left(\frac{775}{5269};-\frac{207}{4222};-\frac{97}{11870}\right)\right\|_\infty=\frac{162}{793}$$

$$\boldsymbol{X^{(4)}=\left(-\frac{1891}{903};\frac{571}{818};-\frac{5011}{765}\right)}\Rightarrow\left\|X^{(4)}-X^{(3)}\right\|_\infty=\left\|\left(-\frac{43}{8421};\frac{45}{26431};\frac{16}{57043}\right)\right\|_\infty=\frac{192}{27083}$$

$$\boldsymbol{X^{(5)}=\left(-\frac{2251}{1075};\frac{2355}{3374};-\frac{976}{149}\right)}\Rightarrow\left\|X^{(5)}-X^{(4)}\right\|_\infty=\left\|\left(\frac{4}{22575};-\frac{7}{117802};\frac{1}{113985}\right)\right\|_\infty=\frac{18}{73355}$$

b)

$$\begin{cases}x_1=\frac{3}{2}-\frac{1}{3}x_2\\ x_2=-\frac{2}{3}+\frac{1}{3}x_3\\ x_3=\frac{5}{4}-\frac{1}{2}x_1\end{cases}$$

$$\boldsymbol{X^{(1)}=(0;0;0)}$$

$$\boldsymbol{X^{(1)}=\left(\frac{3}{2};-\frac{2}{3};\frac{1}{2}\right)}\Rightarrow\left\|X^{(1)}-X^{(0)}\right\|_\infty=\frac{8}{3}$$

$$\boldsymbol{X^{(2)}=\left(\frac{31}{8};-\frac{1}{2};\frac{7}{18}\right)}\Rightarrow\left\|X^{(2)}-X^{(1)}\right\|_\infty=\frac{263}{72}$$

$$\boldsymbol{X^{(3)}=\left(\frac{5}{3};-\frac{29}{54};\frac{5}{12}\right)}\Rightarrow\left\|X^{(3)}-X^{(2)}\right\|_\infty=\frac{491}{216}$$

$$\boldsymbol{X^{(4)}=\left(\frac{136}{81};-\frac{19}{36};\frac{133}{324}\right)}\Rightarrow\left\|X^{(4)}-X^{(3)}\right\|_\infty=\frac{1}{36}$$

$$\boldsymbol{X^{(5)}=\left(\frac{181}{108};-\frac{515}{972};\frac{89}{216}\right)}\Rightarrow\left\|X^{(5)}-X^{(4)}\right\|_\infty=\frac{13}{1944}$$

$$\boldsymbol{X^{(6)}=\left(\frac{1742}{1039};-\frac{343}{648};\frac{345}{838}\right)}\Rightarrow\left\|X^{(6)}-X^{(5)}\right\|_\infty=\frac{30}{19441}$$

$$\boldsymbol{X^{(7)}=\left(\frac{1601}{955};-\frac{1322}{2497};\frac{1601}{3888}\right)}\Rightarrow\left\|X^{(7)}-X^{(6)}\right\|_\infty=\frac{5}{13441}$$

c)

$$\begin{cases} 6x_1 + x_2 + x_3 = 9 \\ x_1 + 6x_2 + 2x_3 = 15 \\ x_1 + x_2 - 6x_3 = -3 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{3}{2} - \frac{1}{6}x_2 - \frac{1}{6}x_3 \\ x_2 = \frac{5}{2} - \frac{1}{6}x_1 - \frac{1}{3}x_3 \\ x_3 = \frac{1}{2} + \frac{1}{6}x_1 + \frac{1}{6}x_2 \end{cases}$$

$$\boldsymbol{X^{(1)} = (0;0;0)}$$

$$\boldsymbol{X^{(1)} = \left(\frac{3}{2};\frac{9}{4};\frac{9}{8}\right)} \Rightarrow \left\|X^{(1)} - X^{(0)}\right\|_\infty = \frac{39}{8}$$

$$\boldsymbol{X^{(2)} = \left(\frac{15}{16};\frac{63}{32};\frac{63}{64}\right)} \Rightarrow \left\|X^{(2)} - X^{(1)}\right\|_\infty = \frac{63}{64}$$

$$\boldsymbol{X^{(3)} = \left(\frac{129}{128};\frac{513}{256};\frac{513}{512}\right)} \Rightarrow \left\|X^{(3)} - X^{(2)}\right\|_\infty = \frac{63}{512}$$

$$\boldsymbol{X^{(4)} = \left(\frac{1023}{1024};\frac{4095}{2048};\frac{4095}{4096}\right)} \Rightarrow \left\|X^{(4)} - X^{(3)}\right\|_\infty = \frac{63}{4096}$$

$$\boldsymbol{X^{(5)} = \left(\frac{8193}{8192};\frac{32769}{16384};\frac{32769}{32768}\right)} \Rightarrow \left\|X^{(5)} - X^{(4)}\right\|_\infty = \frac{63}{32768}$$

$$\boldsymbol{X^{(6)} = \left(\frac{65535}{65536};\frac{262143}{131072};\frac{262143}{262144}\right)} \Rightarrow \left\|X^{(6)} - X^{(5)}\right\|_\infty = \frac{63}{262144}$$

10-

Primero buscamos que la matriz de coeficientes sea DD para asegurar la convergencia, modificando las filas de la misma:

$$\begin{cases} 3x_1 + x_2 = 4 \\ x_1 + 2x_2 = 3 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{4}{3} - \frac{1}{3}x_2 \\ x_2 = \frac{3}{2} - \frac{1}{2}x_1 \end{cases}$$

LA TABLA ESTA EN LA GUÍA

$$\boldsymbol{X^{(0)} = (1,01;1,01);\ X^{(1)} = \left(\frac{299}{300};\frac{199}{200}\right);\ X^{(2)} = \left(\frac{601}{600};\frac{601}{600}\right);\ X^{(3)} = \left(\frac{1799}{1800};\frac{1199}{1200}\right)\ldots(1,1)}$$

Ya vimos que podemos hallar la T_j descomponiendo la matriz de coeficientes o bien usando la matriz de coeficientes obtenida para iterar:

$$T_J = D^{-1}(U+L) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{-\frac{1}{3}} \\ \boldsymbol{-\frac{1}{2}} & \boldsymbol{0} \end{bmatrix}$$

11-

a)

$$\begin{cases} x_1 = 2 - \frac{3}{50}x_2 + \frac{1}{50}x_3 \\ x_2 = 3 - \frac{3}{100}x_1 + \frac{1}{20}x_3 \\ x_3 = 5 - \frac{1}{100}x_1 + \frac{1}{50}x_2 \end{cases}$$

$$\boldsymbol{X^{(1)} = (0;0;0)}$$

$$\boldsymbol{X^{(1)} = (2;3;5)} \Rightarrow \left\|X^{(1)} - X^{(0)}\right\|_\infty = 10$$

$$\boldsymbol{X^{(2)} = (1.920;3.19;5.04)} \Rightarrow \left\|X^{(2)} - X^{(1)}\right\|_\infty = 0{,}31$$

$$\boldsymbol{X^{(3)} = (1.909;3.194;5.045)} \Rightarrow \left\|X^{(3)} - X^{(2)}\right\|_\infty = 0{,}02$$

$$\boldsymbol{X^{(4)} = (1.909;3.195;5.045)} \Rightarrow \left\|X^{(4)} - X^{(3)}\right\|_\infty = 0{,}005$$

b)

$$\begin{cases} x_1 = \frac{10}{3} - \frac{1}{3}x_2 - \frac{1}{3}x_3 \\ x_2 = \frac{21}{5} - \frac{1}{5}x_1 - \frac{2}{5}x_3 \\ x_3 = 6 - \frac{1}{5}x_1 - \frac{2}{5}x_2 \end{cases}$$

$$\boldsymbol{X^{(1)} = (0;0;0)}$$

$$\boldsymbol{X^{(1)} = \left(\frac{10}{3};\frac{21}{5};6\right)} \Rightarrow \left\|X^{(1)} - X^{(0)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(2)} = (-0,067;0,6;3,653)} \Rightarrow \left\|X^{(2)} - X^{(1)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(3)} = (1,916;2,008;5,773)} \Rightarrow \left\|X^{(3)} - X^{(2)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(4)} = (0.74;0.736;4.814)} \Rightarrow \left\|X^{(4)} - X^{(3)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(5)} = (1.483;1.312;5.558)} \Rightarrow \left\|X^{(5)} - X^{(4)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(6)} = (1.044;0.8654;5.179)} \Rightarrow \left\|X^{(6)} - X^{(5)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(7)} = (1.319;1.093;5.445)} \Rightarrow \left\|X^{(7)} - X^{(6)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(8)} = (1.154;0.933;5.3)} \Rightarrow \left\|X^{(8)} - X^{(7)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(9)} = (1.256;1.021;5.396)} \Rightarrow \left\|X^{(9)} - X^{(8)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(10)} = (1.195;0.962;5.341)} \Rightarrow \left\|X^{(10)} - X^{(9)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(11)} = (1.232;0.996;5.376)} \Rightarrow \left\|X^{(11)} - X^{(10)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(12)} = (1.209;0.974;5.355)} \Rightarrow \left\|X^{(12)} - X^{(11)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(13)} = (1.209;0.9743;5.355)} \Rightarrow \left\|X^{(13)} - X^{(11)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(14)} = (1.224;0.987;5.368)} \Rightarrow \left\|X^{(14)} - X^{(12)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(15)} = (1.215;0.979;5.361)} \Rightarrow \left\|X^{(15)} - X^{(14)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(16)} = (1.220;0.983;\ 5.365)} \Rightarrow \left\|X^{(16)} - X^{(15)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(17)} = (1.217\ ;0.981;5.363)} \Rightarrow \left\|X^{(17)} - X^{(16)}\right\|_\infty > 0.01$$

$$\boldsymbol{X^{(18)} = (1.219;0.982;\ 5.364)} \Rightarrow \left\|X^{(18)} - X^{(17)}\right\|_\infty < 0.01$$

12-

En este la caso, la matriz no es DD ni intercambiando las filas, nada podremos asegurar de su convergencia. Es decir, la propiedad de una matriz DD es condición suficiente no necesaria, puede no ser DD y ser convergente.

a)

$$\begin{cases} x_1 = \frac{29}{5} - \frac{7}{5}x_2 \\ x_2 = \frac{79}{20} - \frac{13}{20}x_1 \end{cases}$$

$$X^{(0)} = (1;4);\ X^{(1)} = \left(\frac{1}{5};\frac{191}{50}\right);\ X^{(2)} = \left(\frac{5}{2};\frac{5}{2}\right);\ X^{(3)} = \left(\frac{113}{250};\frac{2329}{637}\right);\ X^{(4)} = \left(\frac{5657}{8303};\frac{491}{140}\right)$$

$$X^{(5)} = \left(\frac{7379}{8291};\frac{1325}{393}\right);\ X^{(6)} = \left(\frac{1311}{1214};\frac{419}{129}\right);\ X^{(7)} = \left(\frac{1849}{1476};\frac{1825}{582}\right);\ X^{(8)} = \left(\frac{1245}{883};\frac{543}{179}\right);$$

$$X^{(9)} = \left(\frac{2429}{1564};\frac{1285}{437}\right);\ X^{(10)} = \left(\frac{675}{401};\frac{6994}{2449}\right);\ X^{(11)} = \left(\frac{3009}{1670};\frac{2048}{737}\right);\ X^{(12)} = \left(\frac{3191}{1671};\frac{279}{103}\right)$$

$$X^{(13)} = \left(\frac{1034}{515};\frac{812}{307}\right);$$

$$X^{(14)} = \left(\frac{1145}{546};\frac{1146}{443}\right);\ X^{(15)} = \left(\frac{1307}{600};\frac{4052}{1599}\right);\ X^{(16)} = \left(\frac{2714}{1205};\frac{711}{286}\right);\ X^{(17)} = \left(\frac{2192}{945};\frac{1375}{563}\right)$$

$$X^{(18)} = \left(\frac{819}{344};\frac{973}{405}\right);\ X^{(19)} = \left(\frac{1747}{717};\frac{1318}{557}\right);\ X^{(20)} = \left(\frac{878}{353};\frac{16270}{6973}\right);\ X^{(21)} = \left(\frac{2541}{1003};\frac{1610}{699}\right)$$

$$X^{(22)} = \left(\frac{4082}{1585};\frac{1204}{529}\right);\ X^{(23)} = \left(\frac{2151}{823};\frac{1461}{649}\right);\ X^{(24)} = \left(\frac{2704}{1021};\frac{3013}{1352}\right);\ X^{(24)} = \left(\frac{3677}{1372};\frac{1826}{827}\right)$$

$$X^{(25)} = \left(\frac{2056}{759};\frac{2406}{1099}\right);\ X^{(26)} = \left(\frac{2147}{785};\frac{1627}{749}\right);\ X^{(27)} = \left(\frac{1476}{535};\frac{1266}{587}\right);\ X^{(28)} = \left(\frac{2205}{793};\frac{1307}{610}\right)$$

$$X^{(29)} = \left(\frac{1711}{611};\frac{1001}{470}\right);\ X^{(30)} = \left(\frac{2187}{776};\frac{3802}{1795}\right);\ X^{(31)} = \left(\frac{3583}{1264};\frac{451}{214}\right);\ X^{(32)} = \left(\frac{3049}{1070};\frac{1051}{501}\right)$$

$$X^{(33)} = \left(\frac{2551}{891};\frac{2887}{1382}\right);\ X^{(34)} = \left(\frac{900}{313};\frac{1259}{605}\right);\ X^{(35)} = \left(\frac{2113}{732};\frac{1238}{597}\right);\ X^{(36)} = \left(\frac{3004}{1037};\frac{2743}{1327}\right)$$

$$X^{(37)} = \left(\frac{619}{213};\frac{439}{213}\right);\ X^{(38)} = \left(\frac{921}{316};\frac{7180}{3493}\right);\ X^{(39)} = \left(\frac{1353}{463};\frac{568}{277}\right);\ X^{(40)} = \left(\frac{621}{212};\frac{1557}{761}\right)$$

$$X^{(41)} = \left(\frac{2234}{761};\frac{1366}{669}\right);\ X^{(42)} = \left(\frac{753}{256};\frac{2087}{1024}\right) \ldots \textit{va hacia } (3,2)$$