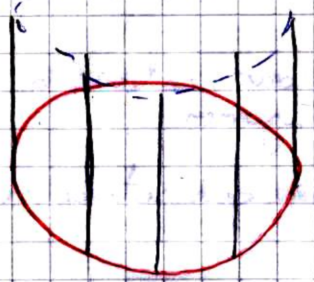
10 Integrales de superficie, flujo

(1) Para tener una idea



La curva directriz es la que da forma a la superficie, no necesariamente debe ser una circunferencia

Las generatrices son las infinitas lineas paralelas que forman el cuerpo de la superficie

Por lo tanto con dichos conceptos se puede determinar facilmente las parametrizaciones.

(a) directriz $y = x^2$ con $z = 0$ generatrices // eje z

Parametrización: $\overline{X}(u,v) = (u, u^2, v)$

(c) " : $\overline{X}(u,v) = (2\cos(u); 2\,\mathrm{sen}(u); v)$

(2) Basicamente lo mismo, pero se pueden aplicar formulas. Hago dos así quedan de ejemplo.

(a) directriz $y = x^2$ $z = 0$ Vertice $(0,0,1)$

El vertice nos sirve para hallar las ecuaciones de las generatrices

$$\frac{X-0}{a} = \frac{X-0}{b} = \frac{Z-1}{c}$$

Como mi directriz esta en el plano XY despejo con Z dividiendo.

$$\begin{cases} \dfrac{X}{Z-1} = \alpha \\ \dfrac{Y}{Z-1} = \beta \end{cases}$$ (a,b,c se convierten en α y β, es un metodo más sencillo)

$$\begin{cases} x = \alpha(z-1) \Longrightarrow = -\alpha \\ \qquad z=0 \Rightarrow \\ y = \beta(z-1) \Longrightarrow = -\beta \end{cases}$$

Ahora reemplazo x e y en la directriz

$y = x^2 \Rightarrow -\beta = \alpha^2$ ya yo se cuanto vale β y α vuelvo a reemplazar

$-\frac{y}{\cancel{z-1}} = \frac{x^2}{(z-1)^{\cancel{2}}} \Rightarrow x^2 = y(1-z)$

$\hookrightarrow$ ecuación cartesiana, en base a ello parametrizamos

$\overline{X} = (u v, u^2 v, 1-v)$ la parametrización tiene que satisfacer la cartesiana.

(c) $x^2+y^2=4 \quad z=0 \quad V(0,0,2)$

$\frac{x}{a} = \frac{y}{b} = \frac{z-2}{c}$

$\begin{cases} \frac{x}{z-2} = \alpha \\ \frac{y}{z-2} = \beta \end{cases} \quad \begin{cases} x = \alpha(z-2) = -2\alpha \\ y = \beta(z-2) = -2\beta \end{cases}$

$(-2\alpha)^2 + (-2\beta)^2 = 4$

$\left(-2\frac{x}{z-2}\right)^2 + \left(-2\frac{y}{z-2}\right)^2 = 4$

$x^2+y^2 = (z-2)^2$ cartesiana

$\overline{X} = (2v\cos(u); 2v\,\mathrm{sen}(u); 2-2v)$

(3) (a) $x^2+y^2+z^2=4$ Es una esfera con ~~rad~~ centro en $\overline{O}$

Su parametrización es $\overline{X} = (2\cos(\alpha)\,\mathrm{sen}(\theta); 2\,\mathrm{sen}(\alpha)\,\mathrm{sen}(\theta); 2\cos(\theta))$

con $\alpha\,[0;\pi] \wedge \theta\,[0;2\pi]$

(b) $x^2+4y^2=4$ Se ve a simple vista para que todo quede igualado

$(2\cos(u))^2 + 4(\mathrm{sen}\,u)^2 = 4$

$4\cos^2(u) + 4\,\mathrm{sen}^2(u) = 4$

$4\underbrace{(\cos^2+\mathrm{sen}^2)}_{1} = 4 \Rightarrow \overline{X} = (2\cos(u); \mathrm{sen}(u), v) \quad u\,[0;2\pi] \quad v \in \mathbb{R}$

como no aparece z se le asigna v que puede variar en cualquier valor real

NOTA

HOJA

FECHA

(c) $X^2+4y^2+9z^2=36$ Hay

$$(6^2 sen^2(\theta)\cos^2(\alpha)) + 4(3^2 sen^2(\theta)\, sen^2(\alpha)) + 9(2^2 \cos^2(\alpha)) = 36$$

$$36\, sen^2(\theta)\cos^2(\alpha) + 36\, sen^2(\theta)\, sen^2(\alpha) + 36\cos^2(\alpha) = 36$$

$$36\underbrace{(sen^2(\theta)\cos^2(\alpha) + sen^2(\theta)\, sen^2(\alpha) + \cos^2(\alpha))}_{1} = 36$$

Es parecido a la esfera solo hay que poner los coeficientes estratégicamente de manera que queden iguales.

$$\overline{X} = (6\, sen(\theta)\cos(\alpha);\ 3\, sen(\theta)\, sen(\alpha);\ 2\cos(\alpha))$$

(d) $\begin{cases} x = u \\ y = v \\ z = u^2+v^2 \end{cases}$ $\quad \overline{X} = (u, v, u^2+v^2)$ con $u, v \in \mathbb{R}$

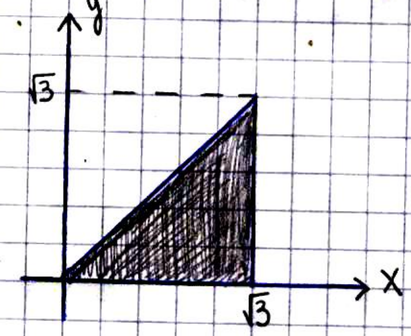
(4) Supongo S def $z = f(x,y)$ (S es proy sobre xy)

$$A(S) = \iint_D \sqrt{\left(-\frac{F'_x}{F'_z}\right)^2 + \left(-\frac{F'_y}{F'_z}\right)^2 + 1}\ dx\,dy = \iint_D \sqrt{\frac{(F'_x)^2+(F'_y)^2+(F'_z)^2}{(F'_z)^2}}$$

$$A(S) = \iint_D \frac{\|\overline{\nabla} F(\overline{x})\|}{|F'_z|}\ dx\,dy$$

(5)(a) Trozo de sup cilíndrica $z = 2x^2$ con $y \le x$ $z \le 6$ 1° octante

Proyecto sobre xy



$$6 = 2x^2 \Rightarrow x = \pm\sqrt{3} \Rightarrow x = \sqrt{3}$$

$$T(x,y) = (x; y; 2x^2) \quad \text{con } 0 \le x \le \sqrt{3},\ 0 \le y \le x$$

$$T'_x \times T'_y = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 1 & 0 & 4x \\ 0 & 1 & 0 \end{vmatrix} = (-4x; 0; 1)$$

$$\|T'_x \times T'_y\| = \sqrt{16x+1}$$

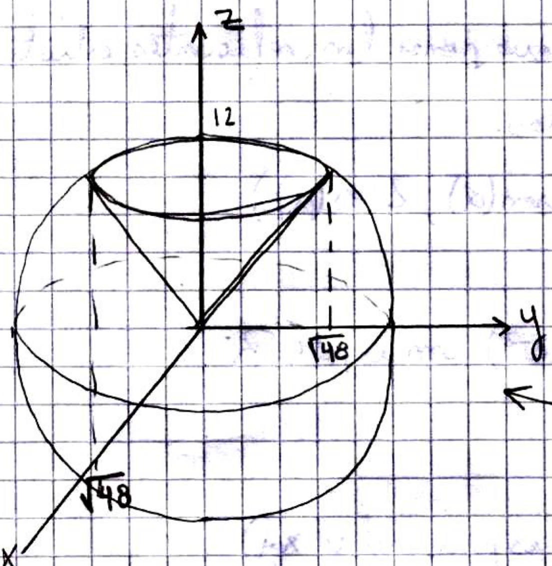
NOTA

$$\int_0^{\sqrt{3}}\int_0^{x} \sqrt{16x^2+1}\,dy\,dx = \int_0^{\sqrt{3}} x\sqrt{16x^2+1}\,dx$$

$u = 16x^2+1 \qquad du = 32x\,dx \qquad \frac{du}{32} = x\,dx$

$$\frac{1}{32}\int_0^{\sqrt{3}} u^{1/2}\,du = \frac{1}{32}\frac{2u^{3/2}}{3}\Bigg|_0^{\sqrt{3}} = \frac{1}{32}\,\frac{2}{3}\,(16(\sqrt{3})^2+1)^{3/2} - \frac{1}{32}\,\frac{2}{3}\,(1)^{3/2} = \boxed{\frac{57}{8}}$$

(b)



$z = \sqrt{2x^2+2y^2}$

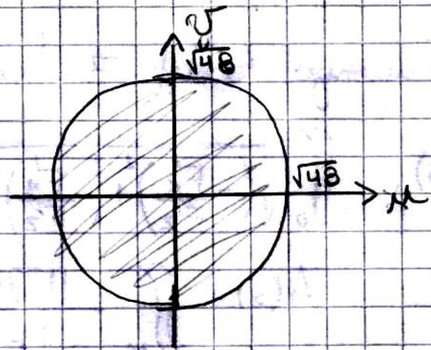
$x^2+y^2+z^2 = 144$

$z = \sqrt{144-x^2-y^2}$

$144 - x^2 - y^2 = 2x^2+2y^2$

$12 = 3x^2+3y^2$

$x^2+y^2 = 48$



$T(u,v) = \left(u, v, \sqrt{2u^2+2v^2}\right)$

$\rightarrow$ Coordenadas polares

$$\begin{cases} x = r\cos(\theta) \\ y = r\,\mathrm{sen}(\theta) \\ z = \sqrt{2}\,r \end{cases}$$

$0 \le r \le \sqrt{48}$

$0 \le \theta \le 2\pi$

NOTA

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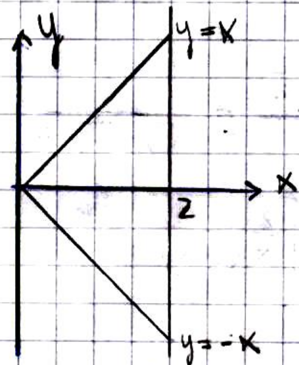
ⓒ Trozo sup cilíndrica $x^2+z^2=4$ con $-x \le y \le x$ $\quad z \geq 0$



$$T(x,y) = (x, y; \sqrt{4-x^2})$$

$$\|\vec{n}\| = \frac{\|(2x; 0, 2z)\|}{|2z|} = \frac{\sqrt{4x^2+4z^2}}{2z} = \frac{\sqrt{4x^2+4(4-x^2)}}{2\sqrt{4-x^2}}$$

$$= \frac{4}{2\sqrt{4-x^2}} = \frac{2}{\sqrt{4-x^2}}$$

$$\int_0^2 \int_{-x}^{x} \frac{2}{\sqrt{4-x^2}}\, dy\, dx = \int_0^2 \frac{2}{\sqrt{4-x^2}}\, 2x\, dx = -4\int_0^2 du = -4\sqrt{4-x^2}\Big|_0^2$$

$$= \boxed{8}$$

$u = \sqrt{4-x^2}$

$du = \frac{-\cancel{2}x}{\cancel{2}\sqrt{4-x^2}}$

ⓓ Superficie esférica radio R

$$T(u,v) = (R\cos u\, \text{sen}\, v; R\, \text{sen}\, u\, \text{sen}\, v; R\cos v) \qquad \begin{cases} 0 \le u \le 2\pi \\ 0 \le v \le \pi \end{cases}$$

$$T'_u \times T'_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -R\,\text{sen}\, u\, \text{sen}\, v & R\cos u\, \text{sen}\, v & 0 \\ R\cos u \cos v & R\,\text{sen}\, u \cos v & -R\,\text{sen}\, v \end{vmatrix}$$

$$= (-R^2\cos u\, \text{sen}^2 v; -R^2\,\text{sen}\, u\, \text{sen}^2 v; -R^2\text{sen}^2 u\, \text{sen}\, v \cos v - R^2\cos^2 u \cos v\, \text{sen}\, v)$$

$$T'_u \times T'_v = -R^2\,\text{sen}\, v(\cos u\, \text{sen}\, v; \text{sen}\, u\, \text{sen}\, v, \cos v) - R^2\,\text{sen}\, v \cos v$$

$$\|T'_u \times T'_v\| = R^2\,\text{sen}\, v \sqrt{\underbrace{\cos^2 u\, \text{sen}^2 v + \text{sen}^2 u\, \text{sen}^2 v}_{\text{sen}^2 v} + \cos^2 v} = R^2\,\text{sen}\, v$$

$$A(S) = \int_0^{2\pi}\int_0^{\pi} R^2\,\text{sen}\, v\, dv\, du = 2\pi R^2(-\cos v)\Big|_0^{\pi} = \boxed{4\pi R^2}$$

NOTA

(e) $x^2+y^2=2x$ con $x^2+y^2+z^2\leq 4$ 1° octante

Proyecto sobre xz

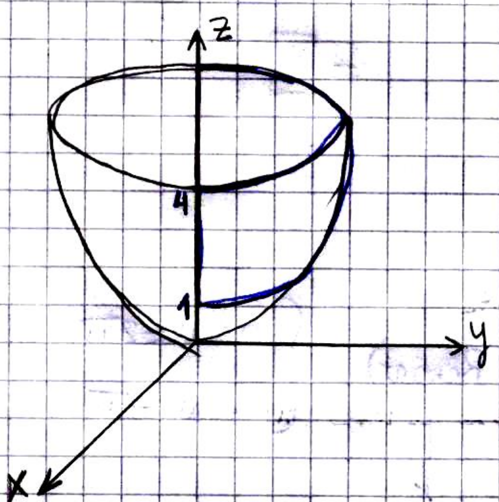
$g(x,y)=(x;\sqrt{2x-x^2};z)$ con $0\leq x\leq 2$ $0\leq z\leq\sqrt{4-2x}$

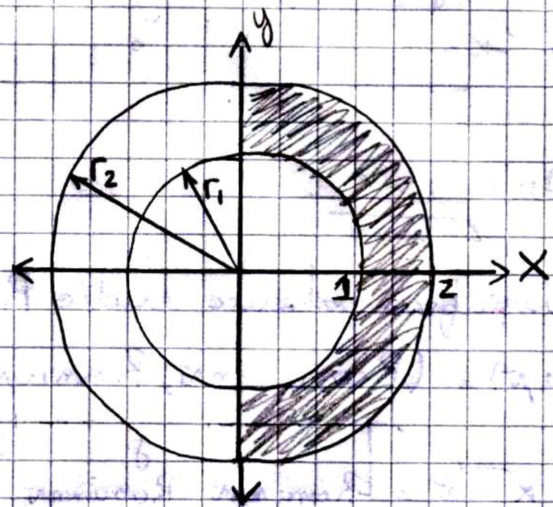
$$\|\check{n}\| = \frac{\|(2x-2;2y;0)\|}{|2y|} = \frac{\sqrt{4x^2-8x+4+8x-4x^2}}{2\sqrt{2x-x^2}} = \frac{1}{\sqrt{2x-x^2}}$$

$$\int_0^2\int_0^{\sqrt{4-2x}} \frac{1}{\sqrt{2x-x^2}}\,dz\,dx = \int_0^2 \frac{\sqrt{4-2x}}{\sqrt{2x-x^2}}\,dx = \int_0^2\sqrt{\frac{2(2-x)}{x(2-x)}}\,dx$$

$$= \int_0^2 \sqrt{2}\,\frac{1}{\sqrt{x}}\,dx = \sqrt{2}\,(2\sqrt{x})\Big|_0^2 = \boxed{4}$$

(6)





Como $x\geq 0$ solo se integra la parte derecha del plano xy

$z=x^2+y^2$ en cilíndricos $z=r^2 \Rightarrow 1\leq z\leq 4 \Rightarrow 1\leq r^2\leq 4$

$1\leq r\leq 2$

$I_z = \iint r^2\,dm = \iint r^2\,\delta\,ds$ $\qquad \delta=\frac{k}{x^2+y^2}=\frac{k}{r^2}$

$\Rightarrow F(x,y)=(x,y,x^2+y^2) \Rightarrow F(r,\theta)=(r\cos\theta;r\,\text{sen}\,\theta;r^2)$

$$ds = F'_r\times F'_\theta = \begin{vmatrix} \check{i} & \check{j} & \check{k}\\ \cos\theta & \text{sen}\,\theta & 2r\\ -r\,\text{sen}\,\theta & r\cos\theta & 0\end{vmatrix} = (-2r^2\cos\theta;-2r^2\text{sen}\,\theta;r\cos^2\theta+r\,\text{sen}^2\theta)$$

$$=(-2r^2\cos\theta;-2r^2\text{sen}\,\theta;r)$$

$$\|ds\| = \sqrt{(-2r^2\cos\theta)^2+(-2r^2\text{sen}\,\theta)^2+r^2} = r\sqrt{4r^2+1}$$

NOTA

HOJA

FECHA

$$I_z = \iint \cancel{r^2}\,\frac{k}{\cancel{r^2}}\; r\sqrt{4r^2+1}\; dr\,d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\pi/2}\int_1^2 k\,r\sqrt{4r^2+1}\; dr\,d\theta$$

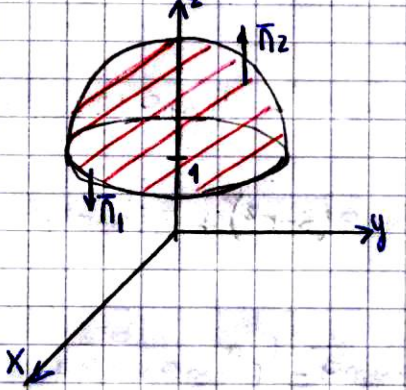
$4r^2+1 = du$

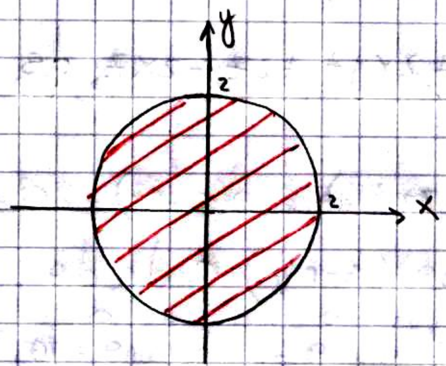
$du = 8r\,dr \qquad \frac{1}{8}du = r\,dr$

$$= k\int_{-\pi/2}^{\pi/2} d\theta \int_1^2 \frac{1}{8}\,u^{1/2}\,dr = \frac{k}{8}\int_{-\pi/2}^{\pi/2}\frac{2}{3}(4r^2+1)^{3/2}\Big|_1^2 = \boxed{\frac{k\pi}{12}\left(17^{3/2}-5^{3/2}\right)}$$

7, 8, 9 muy teóricos, se los debo.

10 a) $f(x,y,z) = (x^2+yz\,;\,xz\,,\,2z^2-2xz)$ a través de la superficie frontera de $1 \le z \le 5-x^2-y^2$





$$\iint_S \bar{f}\,\bar{n}\,ds = \iint_{S_1} f\,\bar{n}_1\,ds + \iint_{S_2} f\,\bar{n}_2\,ds$$

Para S_1:

$$f(\theta,r) = \begin{cases} x = r\cos\theta \\ y = r\,\mathrm{sen}\,\theta \\ z = 1 \end{cases} \qquad |Df| = r \qquad \begin{cases} 0 \le r \le 2 \\ 0 \le \theta \le 2\pi \end{cases}$$

$$n_1 = \frac{\nabla S_1}{|S'_{1z}|} = \frac{(0;0;-1)}{|-1|} = (0,0,-1)$$

NOTA

$$\iint_{S_1} f\, n_1\, ds = \iint_{S_1} (x^2+yz, xz, 2z^2-2xz)(0,0,-1)\, ds = \iint_{S_1} -2z^2+2xz\, ds$$

$$\int_0^{2\pi}\int_0^{2} (-2 + 2r\cos\theta)\, r\, dr\, d\theta = \int_0^{2\pi}\int_0^{2} -2r+2r^2\cos\theta\, dr\, d\theta$$

$$\int_0^{2\pi} -r^2 + \frac{2}{3}r^3\cos\theta \Big|_0^2 d\theta = \int_0^{2\pi} -4+\frac{16}{3}\cos\theta\, d\theta = -4\theta + \frac{16}{3}\operatorname{sen}\theta \Big|_0^{2\pi}$$

$$= \boxed{-8\pi}$$

$25-10r^2+r^4$

Para S_2:

$$\begin{cases} x = r\cos\theta \\ y = r\operatorname{sen}\theta \\ z = 5-r^2 \end{cases} \quad \begin{cases} 0 \le r \le 2 \\ 0 \le \theta \le 2\pi \end{cases} \quad n_2 = \frac{(2x, 2y, 1)}{1} = (2x; 2y, 1)$$

$$= (2r\cos\theta; 2r\operatorname{sen}\theta, 1)$$

$$\iint_{S_2} (x^2+yz, xz; 2z^2-2xz)(2x, 2y, 1)\, ds$$

$$\iint_{S_2} 2x^3+2xyz+2xyz+2z^2-2xz\, ds$$

$$\iint_{S_2} \left(2r^3\cos^3\theta + 2r^2\cos\theta\operatorname{sen}\theta(5-r^2) + 2r^2\cos\theta\operatorname{sen}\theta(5-r^2) + 2(5-r^2)^2 - 2(r\cos\theta)(5-r^2)\right) r\, dr\, d\theta$$

$$\iint_{S_2} [2r^3\cos^3\theta + 10r^2\cos\theta\operatorname{sen}\theta - 2r^3\cos\theta\operatorname{sen}\theta + 10r^2\cos\theta\operatorname{sen}\theta - 2r^4\cos\theta\operatorname{sen}\theta + 50 - 20r^2 + 2r^4 - 10r\cos\theta + 2r^3\cos\theta]\, r$$

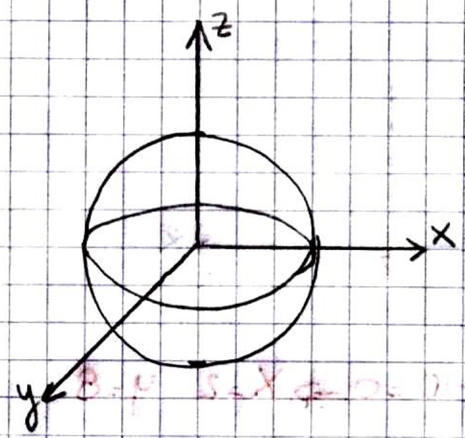
$$\iint_{S_2} 2r^4\cos^3\theta + 10r^3\cos\theta\operatorname{sen}\theta - 2r^4\cos\theta\operatorname{sen}\theta + 10r^3\cos\theta\operatorname{sen}\theta - 2r^5\cos\theta\operatorname{sen}\theta + 50r - 20r^3 + 2r^5 - 10r^2\cos\theta + 2r^4\cos\theta]\, dr\, d\theta = \frac{248\pi}{3}$$

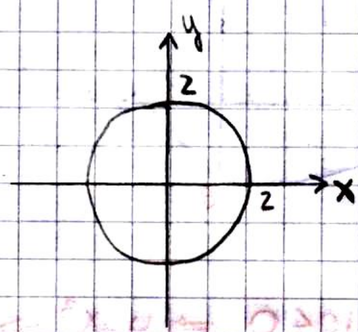
HECHO EN MATH CAD !!!

$$\iint_S f\,\bar{n}\, ds = \frac{248\pi}{3} - 8\pi = \boxed{\frac{224\pi}{3}}$$

NOTA

b) $\overline{F}(x,y,z) = (x,y,z)$ a través de $x^2+y^2+z^2=4$





Para usar un solo $\overline{n}$ vamos a calcular la mitad superior de la esfera y multiplicarlo por dos

$$\iint_S f\overline{n}\,ds = 2\iint_{S'} f\overline{n}\,ds$$

$$\begin{cases} x = r\cos\theta \\ y = r\,\mathrm{sen}\,\theta \\ z = \sqrt{4-r^2} \end{cases} \qquad \begin{cases} 0 \le r \le 2 \\ 0 \le \theta \le 2\pi \end{cases} \qquad \overline{n} = \frac{(2x, 2y; 2z)}{2z}$$

$$\overline{n} = \frac{(\cancel{2}r\cos\theta; 2r\,\mathrm{sen}\,\theta, \cancel{2}\sqrt{4-r^2})}{\cancel{2}\sqrt{4-r^2}}$$

$$\iint_{S'} f\overline{n}\,ds = \int_0^{2\pi}\int_0^2 (r\cos\theta; r\,\mathrm{sen}\,\theta; \sqrt{4-r^2})\,\frac{(r\cos\theta, r\,\mathrm{sen}\,\theta, r\sqrt{4-r^2})}{\sqrt{4-r^2}}\,|Df|\,dr\,d\theta$$

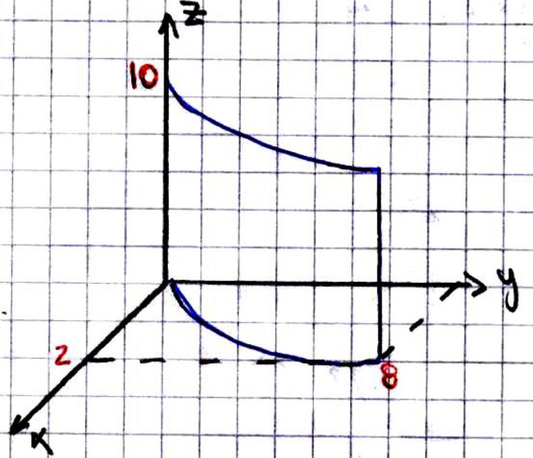
$$= \int_0^{2\pi}\int_0^2 \frac{r^3\cos^2\theta + r^3\,\mathrm{sen}^2\theta + r(4-r^2)}{\sqrt{4-r^2}}\,dr\,d\theta = \int_0^{2\pi}\int_0^2 \frac{\cancel{r^3} + 4r - \cancel{r^3}}{\sqrt{4-r^2}}\,dr\,d\theta$$

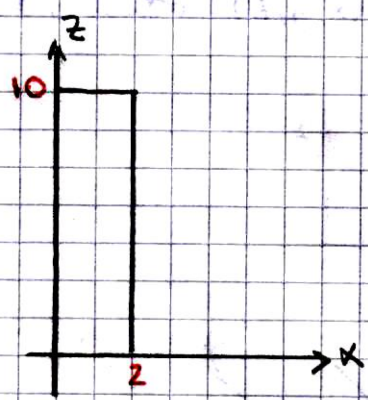
$$4\int_0^{2\pi}\int_0^2 \frac{r}{\sqrt{4-r^2}}\,dr\,d\theta = 16\pi$$ Hecha por PC

$$\longrightarrow \boxed{\iint_S f\overline{n}\,ds = 32\pi}$$

NOTA

(c) $\overline{f}(x,y,z) = (xy, zx, y - xz^2)$ $\quad y = x^3$ con $0 \le z \le x+y \quad x+y \le 10$





$$x+y-10 \le 0 \Rightarrow y=x^3 \Rightarrow x+x^3-10=0 \Rightarrow x=2 \quad y=8$$

$$f(x,z) = (x, x^3; z) \qquad \begin{cases} 0\le x\le 2 \\ 0\le z\le x+x^3\end{cases}$$

$$\overline{n} = f'_x \times f'_z = \begin{vmatrix} i & j & k\\ 1 & 3x^2 & 0 \\ 0 & 0 & 1\end{vmatrix} = (3x^2, -1, 0)$$

$$\iint_S f\,\overline{n}\,ds = \iint_S (x^4, xz, x^3 - xz^2)(3x^2, -1; 0)\,ds$$

$$\iint_S 3x^6 - xz\,ds = \int_0^2\int_0^{x+x^3} 3x^6 - xz\,dz\,dx = \int_0^2 3x^6 z - \frac{x}{2}z^2 \Big|_0^{x+x^3} dx$$

$$\int_0^2 3x^6(x+x^3) - \frac{x(x+x^3)^2}{2}\,dx = \int_0^2 3x^7+3x^9 - \frac{x}{2}(x^6+2x^4+x^2)\,dx$$

$$\int_0^2 3x^7+3x^9-\frac{x^7}{2}-x^5-\frac{x^3}{2}\,dx = \int_0^2 \frac{5}{2}x^7+3x^9-x^5-\frac{x^3}{2}\,dx$$

$$\frac{5}{2}\frac{1}{8}x^8 + \frac{3}{10}x^{10} - \frac{x^6}{6} - \frac{x^4}{8}\Big|_0^2 = \boxed{\frac{5618}{15}}$$

NOTA

HOJA

FECHA

(11) $\bar{f}(x,y,z) = (x-y\,;x-z;\,g(x,y,z))$ $\quad x=y^2$ con $0\le z\le 4$ $\quad 0\le y\le 2-x^2$

Parametrizo S $\qquad u = 2-u^4 \;/\; u=1$

$$\begin{cases} x = u^2 \\ y = u \\ z = v \end{cases} \quad \bar{T}(u,v) = (u^2;u;v) \quad D:\begin{cases} 0\le u\le 1 \\ 0\le v\le 4 \end{cases}$$

$$\iint \bar{f}\,\bar{n}\,ds = \iint_D \bar{f}(\bar{T}(u,v))(T'_u \times T'_v)\,du\,dv$$

$$T'_u \times T'_v = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 2u & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (1;-2u;0)$$

como es $=0$ cuando multiplique $g(x,y,z)$ se cancela por ende no influye.

$$\iint [(u^2-u).1+(u^2-v)(-2u)]\,du\,dv = \iint u^2-u+2uv-2u^3\,dv\,du$$

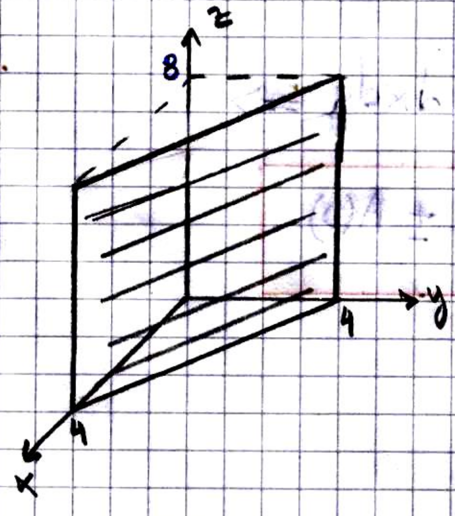
$$\int_0^1\int_0^4 -2u^3+u^2-u+2uv\,dv\,du = \int_0^1 -2u^3v+u^2v-uv+uv^2\Big|_0^4\,du$$

$$\int_0^1 -8u^3+4u^2-4u+16u\,du = \int_0^1 -8u^3+4u^2+12u\,du$$

$$= -\frac{8}{4}u^4+\frac{4}{3}u^3+6u^2\Big|_0^1 = -2+\frac{4}{3}+6 = \boxed{\frac{16}{3}}$$ con normal hacia plano xz

(12) $f(x,y,z) = x+y+z\,g(x-y)$

$$\bar{n} = \nabla f(x,y,z) = (1+z\,g'(x-y);1-z\,g'(x-y);g(x-y))$$



$$\begin{cases} x = u \\ y = 4-u \\ z = v \end{cases} \quad T(u,v) = (u; 4-u; v) \quad D: \begin{cases} 0 \leq u \leq 4 \\ 0 \leq v \leq 8 \end{cases} \Rightarrow \boxed{0 \leq z \leq 8}$$

$$T'_u \times T'_v = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (-1; -1; 0)$$

Reemplazo $T(u,v)$ en $\bar{f}$

$$\iint_D (1 + v\,g'(2u-4); 1 + v\,g'(2u-4); g(2u-4))(-1;-1;0) \cdot$$

$$\iint -1 - \cancel{v\,g'(2u-4)} + 1\,\cancel{v\,g'(2u-4)}\,du\,dv$$

$$-2 \underbrace{\iint_D du\,dv}_{A(D)} = -2 \times 32 = -64 \text{ con sentido con normales componentes negativas.}$$

(13) $g(\bar{r}) = g(x,y,z) = z + h(x,y) = z + h(w)$

$$w = x \cdot y \Rightarrow \frac{\partial w}{\partial x} = y \quad \frac{\partial w}{\partial y} = x$$

$$\Rightarrow \nabla g = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) g = \left(\underbrace{\frac{\partial h}{\partial w}}_{h'} \cdot \underbrace{\frac{\partial w}{\partial x}}_{y}, \underbrace{\frac{\partial h}{\partial w}}_{h'} \underbrace{\frac{\partial w}{\partial y}}_{x}; 1\right)$$

$$\overline{\nabla} g = (h'y; h'x; 1)$$

$T = (x, y, y^2 - x^2) = T(x,y)$ Parametrización de $z = y^2 - x^2$

$$\underbrace{T'_x \times T'_y}_{ds} = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 1 & 0 & -2x \\ 0 & 1 & 2y \end{vmatrix} = (2x; -2y; 1)$$

$$\overline{\nabla} g \cdot (T'_x \times T'_y) = (2xy\,h' - 2xy\,h' + 1)\,dx\,dy \Rightarrow$$

$$\boxed{\iint \overline{\nabla} g\,d\bar{s} = \iint dx\,dy = \pm A(D)}$$

NOTA

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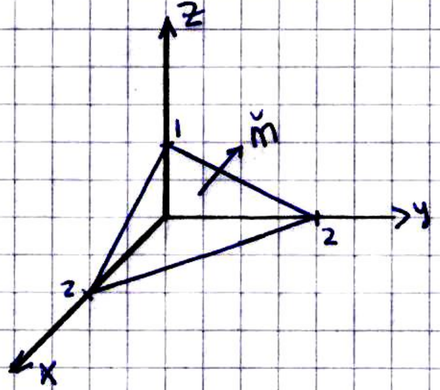
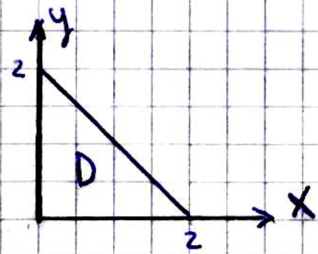
FECHA

(15) $\bar{f}(x,y,z) = (x, 2y; z)$ $\quad x+y+2z=2$ $\quad u+v+1-\frac{u}{2}-\frac{v}{2}$

$\frac{x}{2}+\frac{y}{2}+z=1$ $\quad \frac{1}{2}u+\frac{1}{2}v=$





$$\begin{cases} x = u \\ y = v \\ z = 1-\frac{u}{2}-\frac{v}{2} \end{cases} \qquad T(u,v) = \left(u, v, 1-\frac{u}{2}-\frac{v}{2}\right) \qquad D: \begin{cases} 0 \le u \le 2 \\ 0 \le v \le -u+2 \end{cases}$$

$$T'_u \times T'_v = \begin{vmatrix} \breve{i} & \breve{j} & \breve{k} \\ 1 & 0 & -1/2 \\ 0 & 1 & -1/2 \end{vmatrix} = (1/2; 1/2; 1)$$

$$\iint_S f\breve{m}\,ds = \iint_D \left(u, 2v; 1-\frac{u}{2}-\frac{v}{2}\right)(1/2; 1/2; 1)\,du\,dv$$

$$= \iint_D \cancel{\frac{u}{2}} + \frac{v}{2} + 1 - \cancel{\frac{u}{2}} - \frac{v}{2} = \iint_D 1 + \frac{1}{2}v\,du\,dv$$

$$\int_0^2 \int_0^{2-u} 1 + \frac{1}{2}v\,dv\,du = \int_0^2 v + \frac{1}{4}v^2 \Big|_0^{2-u} du = \int_0^2 2-u+\frac{1}{4}(4-4u+u^2)\,du$$

$$\int_0^2 \frac{u^2}{4} - u + 1 - u + 2\,du = \int_0^2 \frac{u^2}{4} - 2u + 3\,du$$

$$\frac{u^3}{12} - u^2 + 3u \Big|_0^2 = \boxed{\frac{8}{3}}$$

NOTA