Teoría de los Circuitos I

Universidad Tecnológica Nacional

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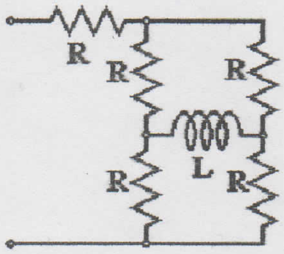
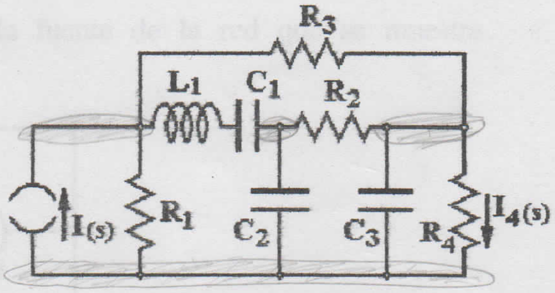
Guía de Trabajos Prácticos N° 8

Resolución Sistemática de Circuitos

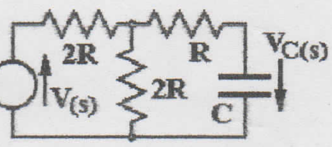
1. En el circuito de la figura calcular la impedancia compleja de excitación aplicando:
a) El método de los nodos.
b) El método de las mallas.
Datos: R = 1 kΩ; L = 1 Hy; $\omega = 10^3$ 1/s.



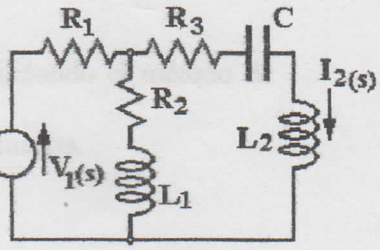
2. En el circuito de la figura, se pide calcular:
a) La corriente $I_4(s)$ aplicando el método de los nodos para una I(s) genérica.
b) ¿Qué sucedería si en vez de un generador de corriente tuviéramos un generador de tensión?



3. Para el circuito de la figura calcular:
a) La tensión $V_C(s)$ aplicando el método de las mallas para una excitación V(s) genérica.
b) Calcular $V_C(t)$ para v(t) = 5V.e-2.t.u(t).
Datos: R = 1 kΩ ; C = 1 μF

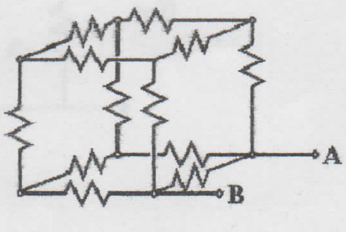


4. Dado el siguiente circuito calcular la impedancia de transferencia $Z_T(s) = \left.\frac{V_1(s)}{I_2(s)}\right]_{CIN}$ por aplicación del método de los nodos.

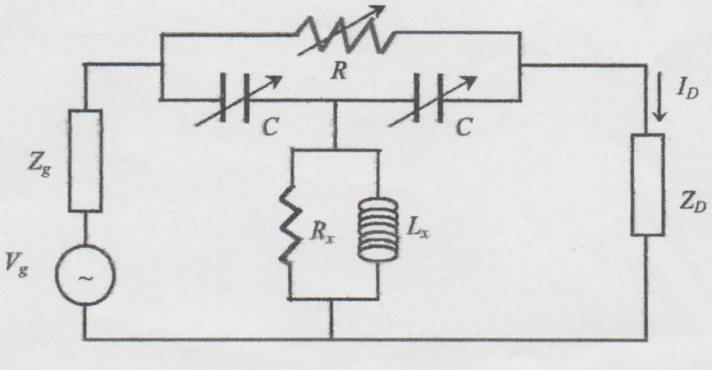


hacer MatLab

5. En la siguiente red espacial de resistores de 1 Ω, calcular el valor de la resistencia entre los puntos A y B aplicando el método de los nodos. Utilice *MatLab* para resolver la matriz de resistencias obtenida.

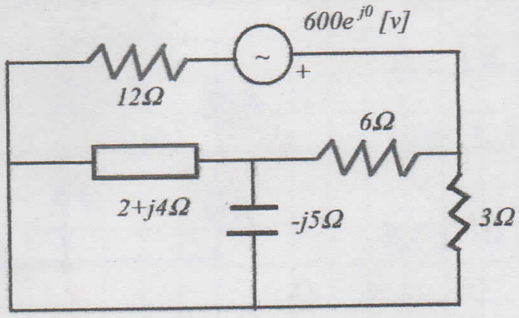


6. En la red mostrada en el siguiente circuito, las dos capacidades iguales C y la resistencia de derivación R se ajustan hasta que la corriente I_D en el detector es cero. Suponiendo una

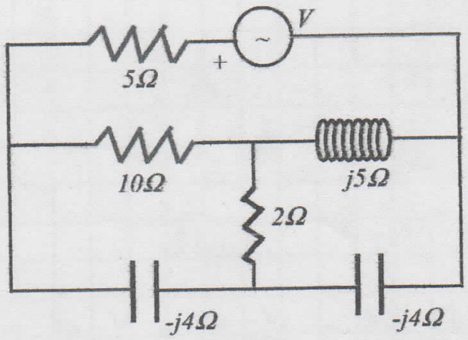


frecuencia angular de la fuente ω constante, determine los valores de R_x y L_x.

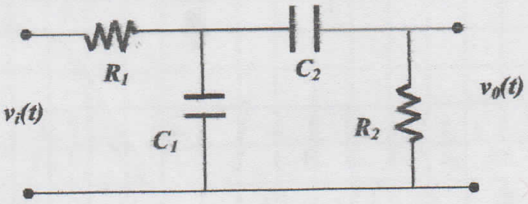
7. Para el circuito de la siguiente figura, se pide:
   a) Plantear las ecuaciones de nodos para el circuito que se muestra a continuación.
   b) Determinar la corriente que circula por la resistencia de 6Ω y su caída de tensión.



8. Determine la impedancia de entrada vista desde la fuente de la red que se muestra continuación:



9. Para el circuito que se muestra en el figura, se pide:
   a) Calcular la función de transferencia del sistema, es decir *H(s)*, utilizando el método de nodos.
   b) Verifique lo calculado en el inciso anterior utilizando el método de mallas.



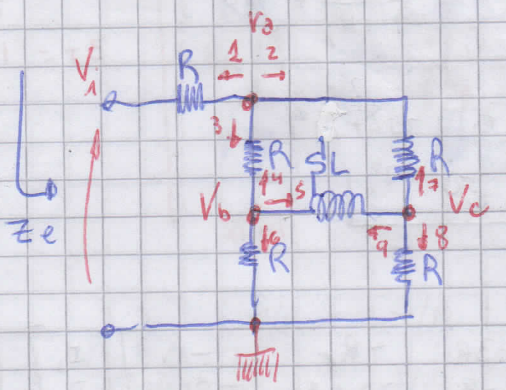
Guía de Trabajos Prácticos N° 8 | HOJA N°
Resolución Sistemática de Circuitos | FECHA

1) Calcular la impedancia compleja de exitación

a) Nodos b) mallas

$I = \frac{V_a}{R} - \frac{V}{R}$

$Z_e(s) = \frac{V(s)}{I(s)}$



$R = 1K\Omega$

$L = 1Hy$

$W = 10^3 \frac{1}{seg}$

$Y \cdot V = I$

a) Nodos: [Todas las corrientes entrantes o salientes]

$$\begin{bmatrix} \frac{3}{R} & -\frac{1}{R} & -\frac{1}{R} \\ -\frac{1}{R} & \frac{1}{R}+\frac{1}{R}+\frac{1}{sL} & -\frac{1}{sL} \\ -\frac{1}{R} & -\frac{1}{sL} & \frac{1}{R}+\frac{1}{R}+\frac{1}{sL} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} \frac{V}{R} \\ 0 \\ 0 \end{bmatrix}$$

- $I_1 + I_2 + I_3 = 0 = \frac{V_a - V}{R} + \frac{V_a - V_c}{R} + \frac{V_a - V_b}{R} \Rightarrow \frac{3V_a}{\cancel{R}} - \frac{V_b}{\cancel{R}} - \frac{V_c}{\cancel{R}} = \frac{V}{\cancel{R}}$

- $I_4 + I_5 + I_6 = 0 = \frac{V_b - V_a}{R} + \frac{V_b - V_c}{sL} + \frac{V_b}{R} = 0 \Rightarrow -\frac{V_a}{R} + \left(\frac{2}{R} + \frac{1}{sL}\right)V_b - \frac{V_c}{sL} = 0$

- $I_7 + I_8 + I_9 = 0 = \frac{V_c - V_a}{R} + \frac{V_c}{R} + \frac{V_c - V_b}{sL} = 0 \Rightarrow -\frac{V_a}{R} - \frac{V_b}{sL} + \left(\frac{2}{R} + \frac{1}{sL}\right)V_c = 0$

$Y \cdot V = I$

$\Delta_Y = DY(s) = \frac{3}{R}\left[\left(\frac{2}{R}+\frac{1}{sL}\right)^2 - \left(\frac{1}{sL}\right)^2\right] + \frac{1}{R}\left[-\frac{1}{R}\left(\frac{2}{R}+\frac{1}{sL}\right) - \frac{1}{sLR}\right] - \frac{1}{R}\left[\frac{1}{RsL} + \frac{1}{R}\left(\frac{2}{R}+\frac{1}{sL}\right)\right]$

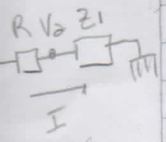
$DY(s) = \frac{3}{R}\left(\frac{4}{R^2} + \frac{4}{sLR} + \cancel{\frac{1}{(sL)^2}} - \cancel{\frac{1}{(sL)^2}}\right) + \frac{1}{R}\left(-\frac{2}{R^2} - \frac{2}{sLR}\right) - \frac{1}{R}\left(\frac{1}{sRL} + \frac{2}{R^2} + \frac{1}{sRL}\right)$

$DY(s) = \frac{12}{R^3} + \frac{12}{sLR^2} - \frac{2}{R^3} - \frac{2}{sLR^2} - \frac{2}{sLR^2} - \frac{2}{R^3} = \frac{8}{sLR^2} + \frac{8}{R^3}$

$\boxed{DY(s) = \frac{8R + 8sL}{sLR^3}}$

$\frac{4sL + 4R}{sLR^2}$

$V_a = \frac{Det\begin{bmatrix} V/R & -1/R & -1/R \\ 0 & \frac{2}{R}+1/sL & -1/sL \\ 0 & -1/sL & \frac{2}{R}+1/sL \end{bmatrix}}{\Delta Y} = \frac{\frac{V}{R}\left[\left(\frac{2}{R}+\frac{1}{sL}\right)^2 - \frac{1}{(sL)^2}\right]}{\Delta Y} \rightarrow \frac{V_a}{I} = \frac{\frac{4}{R^2} + \frac{4}{sLR}}{\frac{8R + 8sL}{sLR^3}}$



$Z_1 = \frac{V_a}{I}$

$\frac{V_a}{I} = Z_1 \rightarrow Z_T = R + Z_1 = R + \frac{4sL + 4R}{\cancel{s}LR^2} \cdot \frac{\cancel{s}LR^3}{8R + 8sL} = R + R\,\frac{4sL + 4R}{8R + 8sL}$

NOTA

$$Z_T = R + R\,\frac{4R+4\xi L}{8R+8\xi L} = R + R\,\frac{4L}{9L}\,\frac{\xi + \frac{R}{L}}{\xi + \frac{8R}{9L}}$$

$$Z_T = R + \frac{4R}{9}\,\frac{\xi + R/L}{\xi + \frac{8}{9}R/L} = 1K\Omega + \frac{4}{9}.1K\Omega\,\frac{\frac{1K\Omega}{1Hy} + j10^3\frac{1}{s}}{\frac{8}{9}\frac{1K\Omega}{1Hy} + j10^3\frac{1}{s}}$$

$$Z_T = 1K\Omega + \frac{4K}{9}\,1{,}057\,e^{-j3{,}36^\circ}$$

$$Z_T = 1K\Omega\,e^{j0^\circ} + 470\Omega\,e^{-j3{,}36^\circ} = 1467{,}19 - j27{,}54$$

$$\boxed{Z_T = (1467{,}2 - j\,27{,}54)\,\Omega = 1467{,}46\,e^{-j1{,}07^\circ}}$$

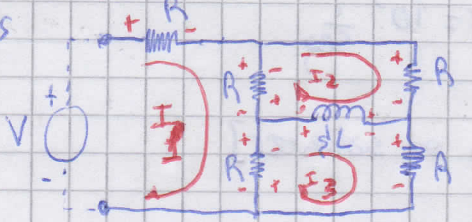
$$R + R\,\frac{4L}{8L}\,\frac{\frac{R}{L}+\xi}{\frac{R}{L}+\xi} \quad 1$$

$$R + \frac{1}{2}R$$

$$Z_T = \frac{3}{2}R$$

$$Z_T = 1500\Omega$$

b) Mallas



$$\begin{bmatrix} 3R & -R & -R \\ -R & 2R+\xi L & -\xi L \\ -R & -\xi L & 2R+\xi L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbb{Z}\,.\,\mathbb{I} = \mathbb{V}$$

$$\Delta Z = 3R\left[(2R+\xi L)^2 - (\xi L)^2\right] + R\left[-2R^2 - \xi LR - \xi LR\right] - R\left[\xi RL + 2R^2 + \xi LR\right]$$

$$\Delta Z = 3R\left((2R)^2 + 4R\xi L + \cancel{(\xi L)^2} - \cancel{(\xi L)^2}\right) - 2R^3 - 4\xi LR^2 - 2R^3$$

$$\boxed{\Delta Z = 12R^3 + 12R^2\xi L - 4R^3 - 4\xi LR^2 = 8R^3 + 8\xi R^2 L = 8R^2L\left(\xi + \frac{R}{L}\right)}$$

$$I_1 = \frac{\operatorname{Det}\begin{bmatrix} V & -R & -R \\ 0 & 2R+\xi L & -\xi L \\ 0 & -\xi L & 2R+\xi L \end{bmatrix}}{\Delta Z} = \frac{V\left[(2R+\xi L)^2 - (\xi L)^2\right]}{\Delta Z}$$

$$\frac{I_1}{V} = \frac{4R^2 + 4R\xi L}{\xi + \frac{R}{L}} \cdot \frac{1}{8R^2L} = \frac{4RL}{8R^2L}\,\frac{\cancel{\xi + \frac{R}{L}}}{\cancel{\xi + \frac{R}{L}}} = \frac{1}{2R}$$

$$V = I_1(R + Z_1)$$

$$\frac{I_1}{V} = \frac{1}{R+Z_1} = \frac{1}{2R} \rightarrow Z_T = R + Z_1 = 2R = 2000\Omega$$

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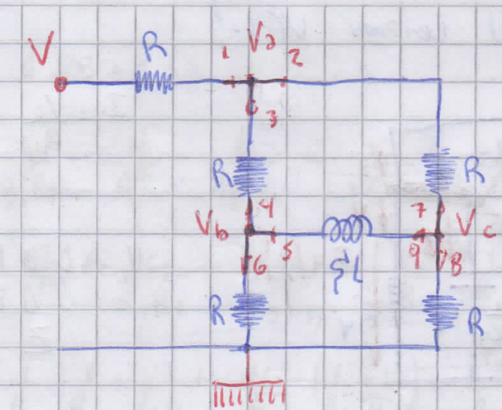
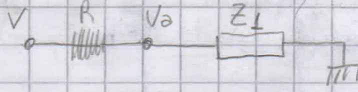
1)



$R = 1K$ $\quad L = 1Hy$ $\quad W = 1 \times 10^3 \frac{1}{s}$

$N \to Y \cdot V = I$ $\qquad M \to Z\, I = V$



$Z_T = R + Z_1$

$I = \frac{V}{R+Z_1} = \frac{V_a}{Z_1} = \frac{V - V_a}{R} = \frac{V}{R} - \frac{V_a}{R}$

Nodos:

$$\begin{bmatrix} \frac{3}{R} & -1/R & -1/R \\ -1/R & \frac{2}{R} + \frac{1}{sL} & -1/sL \\ -1/R & -1/sL & \frac{2}{R} + \frac{1}{sL} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} \frac{V}{R} \\ 0 \\ 0 \end{bmatrix}$$

$Z_T = R + Z_1 = R + \frac{V_a}{I}$

$Z_T = R + \frac{V_a}{\frac{V}{R} - \frac{V_a}{R}}$

$$\Delta Y = \text{Det } Y = \frac{3}{R}\left[\frac{4}{R^2} + 2 \cdot \frac{2}{sRL} + \cancel{\left(\frac{1}{sL}\right)^2} - \cancel{\left(\frac{1}{sL}\right)^2}\right] + \frac{1}{R}\left[-\frac{2}{R^2} - \frac{1}{sRL} - \frac{1}{sRL}\right] - \frac{1}{R}\left[\frac{1}{sRL} + \frac{2}{R^2} + \frac{1}{sRL}\right]$$

$$\Delta Y = \underbrace{\frac{12}{R^3}}_{} + \frac{12}{sR^2L} - \underbrace{\frac{2}{R^3}}_{} - \frac{2}{sR^2L} - \frac{2}{sR^2L} - \underbrace{\frac{2}{R^3}}_{} = \frac{8}{R^3} + \frac{8}{sR^2L} = 8\left(\frac{sL + R}{sR^3L}\right)$$

$$\boxed{\Delta Y = 8\, \frac{s + R/L}{s R^3}}$$

$$V_a = \frac{\text{Det}\begin{bmatrix} V/R & -1/R & -1/R \\ 0 & \frac{2}{R} + \frac{1}{sL} & -1/sL \\ 0 & -1/sL & \frac{2}{R} + \frac{1}{sL} \end{bmatrix}}{\Delta Y} = \frac{\frac{V}{R}\left[\frac{4}{R^2} + 2\,\frac{2}{sRL}\right]}{\Delta Y} \to \frac{V_a}{\frac{V}{R}} = \frac{4\left[\frac{sL + R}{sR^2L}\right]}{\Delta Y}$$

$$\frac{V_a}{\frac{V}{R}} = \frac{1}{2} \cdot \frac{\cancel{s + R/L}}{\cancel{sR^2}} \cdot \frac{\cancel{s}R^{\cancel{3}}}{\cancel{s + R/L}} = \boxed{\frac{1}{2} R}$$

$V = IR + V_a$

$R = \frac{V I R^2}{V} + \frac{V_a}{\frac{V}{R}}$ $\qquad V_a \left(1 + 1\right)$

$R - R^2 \frac{I}{V} = \frac{V_a}{\frac{V}{R}}$

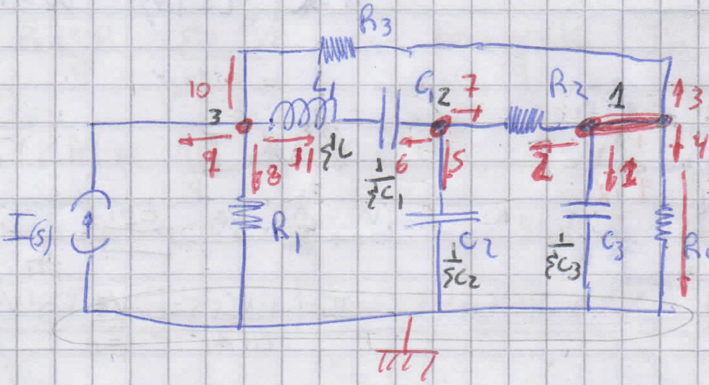
$R - R^2 Y_t = \frac{1}{2} R \to Y_t = \frac{0{,}5R - R}{-R^2} = \frac{R - 0{,}5R}{R^2}$

$Z_T = \frac{R^2}{R - 0{,}5R} = 2000\,\Omega = 2K\Omega$

NOTA

2) a) calcular $I_4(s)$ por Nodos ($I(s)$ genérica)

b) ¿Qué sucede si en vez de $I(s)$ tenemos $V(s)$?



$I_4 = \dfrac{V_1}{R_4}$

$V_1 = \dfrac{\text{Det } M V_1}{\Delta Z}$

$Y \cdot V = I$

$$\begin{bmatrix} sC_3 + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} & -1/R_2 & -1/R_3 \\ -1/R_2 & sC_2 + \frac{1}{sL_1 + \frac{1}{sC_1}} + \frac{1}{R_2} & -\frac{1}{sL_1 + \frac{1}{sC_1}} \\ -1/R_3 & -\frac{1}{sL_1 + \frac{1}{sC_1}} & \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{sL_1 + \frac{1}{sC_1}} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I(s) \end{bmatrix}$$

$$\left(sC_3 + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) \cdot \left[\left(sC_2 + \frac{sC_1}{s^2L_1C_1+1} + \frac{1}{R_2}\right)\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{sC_1}{s^2L_1C_1+1}\right) - \left(\frac{sC_1}{s^2L_1C_1+1}\right)^2\right] =$$

$$= \left(sC_3 + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)\left[\frac{sC_2}{R_1} + \frac{sC_2}{R_3} + \frac{s^2C_1C_2}{s^2L_1C_1+1} + \frac{sC_1}{R_1(s^2L_1C_1+1)} + \frac{sC_1}{R_3(s^2L_1C_1+1)} + \cancel{\left(\frac{sC_1}{s^2L_1C_1+1}\right)^2} + \right.$$

$$\left. + \frac{1}{R_1R_2} + \frac{1}{R_2R_3} + \frac{sC_1}{R_2(s^2L_1C_1+1)} - \cancel{\left(\frac{sC_1}{s^2L_1C_1+1}\right)^2}\right] =$$

A) $$= \left(sC_3 + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)\left[\frac{sC_1}{s^2L_1C_1+1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) + sC_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{s^2C_1C_2}{s^2L_1C_1+1} + \frac{1}{R_1R_2} + \frac{1}{R_2R_3}\right] =$$

B) $$\frac{1}{R_2}\left[-\frac{1}{R_2} \cdot \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{sC_1}{s^2L_1C_1+1}\right) - \frac{1}{R_3}\frac{sC_1}{(s^2L_1C_1+1)}\right] = -\frac{1}{R_2^2R_1} - \frac{1}{R_2^2R_3} - \frac{sC_1}{R_2^2(s^2L_1C_1+1)} - \frac{sC_1}{R_2R_3(s^2L_1C_1+1)}$$

C) $$-\frac{1}{R_3}\left[\frac{sC_1}{R_2(s^2L_1C_1+1)} + \frac{sC_2}{R_3} + \frac{sC_1}{R_3(s^2L_1C_1+1)} + \frac{1}{R_2R_3}\right] = -\frac{sC_1}{R_2R_3(s^2L_1C_1+1)} - \frac{sC_2}{R_3^2} - \frac{sC_1}{R_3^2(s^2L_1C_1+1)} - \frac{1}{R_2R_3^2}$$

$$K = \frac{\xi C_1}{\xi^2 L_1 C_1 + 1} \qquad q = \frac{\xi^2 C_1 C_2}{\xi^2 L_1 C_1 + 1}$$

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FECHA

$$\Delta Y = \bar{A} + B + C \Rightarrow B + C + A$$

$$\Rightarrow \cancel{\frac{-1}{R_2^2 R_1}} - \cancel{\frac{1}{R_2^2 R_3}} - \cancel{\frac{K}{R_2^2}} - \cancel{\frac{K}{R_2 R_3}} - \cancel{\frac{K}{R_2 R_3}} - \cancel{\frac{K}{R_3^2}} - \frac{\xi C_2}{R_3^2} - \cancel{\frac{1}{R_2 R_3^2}} + \xi C_3 \cdot K \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) + \xi^2 C_2 C_3 \left(\frac{1}{R_1} + \frac{1}{R_2}\right) +$$

$$+ \xi C_3 q + \frac{\xi C_3}{R_1 R_2} + \frac{\xi C_3}{R_2 R_3} + \frac{K}{R_2}\left(\frac{1}{R_1} + \cancel{\frac{1}{R_2}} + \cancel{\frac{1}{R_3}}\right) + \frac{\xi C_2}{R_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{q}{R_2} + \cancel{\frac{1}{R_1 R_2^2}} + \cancel{\frac{1}{R_2^2 R_3}} +$$

$$+ \frac{K}{R_3}\left(\frac{1}{R_1} + \cancel{\frac{1}{R_2}} + \cancel{\frac{1}{R_3}}\right) + \frac{\xi C_2}{R_3}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{q}{R_3} + \frac{1}{R_1 R_2 R_3} + \cancel{\frac{1}{R_2 R_3^2}} + \frac{K}{R_4}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) + \frac{\xi C_2}{R_4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) +$$

$$+ \frac{q}{R_4} + \frac{1}{R_1 R_2 R_4} + \frac{1}{R_2 R_3 R_4} =$$

$$= \xi C_2 \Bigg(\overbrace{\frac{1}{R_1 R_3} + \frac{1}{R_2 R_3} + \frac{1}{R_1 R_2} + \frac{1}{R_2^2} + \frac{1}{R_1 R_4} + \frac{1}{R_2 R_4} - \frac{1}{R_3^2}}^{\rho}\Bigg) + K \Bigg(\overbrace{\frac{1}{R_1 R_2} + \frac{1}{R_1 R_3} + \frac{1}{R_1 R_4} + \frac{1}{R_2 R_4} + \frac{1}{R_3 R_4}}^{\theta}\Bigg) +$$

$$+ \xi C_3 \left(\frac{1}{R_1 R_2} + \frac{1}{R_2 R_3}\right) + q \left(\frac{1}{R_4} + \frac{1}{R_3} + \frac{1}{R_2}\right) + \xi C_3 K \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) + \xi^2 C_2 C_3 \left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \xi C_3 q +$$

$$+ \frac{1}{R_1 R_2 R_3} + \frac{1}{R_1 R_2 R_4} + \frac{1}{R_2 R_3 R_4} =$$

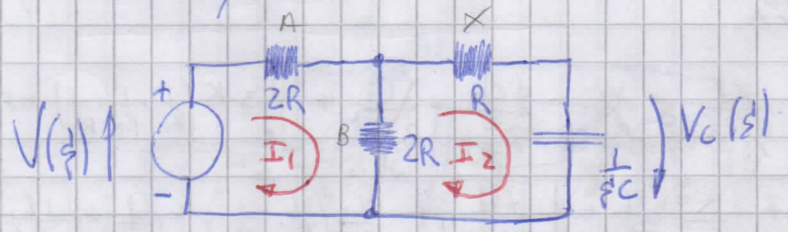
- $$\text{Det } MV_1 = \text{Det} \begin{bmatrix} 0 & -1/R_2 & -1/R_3 \\ 0 & \xi C_2 + K + 1/R_2 & -K \\ I & -K & 1/R_1 + 1/R_3 + K \end{bmatrix} = \frac{K}{R_2} I + \frac{1}{R_3}\left[+I\left(\xi C_2 + K + 1/R_2\right)\right]$$

$$\Delta V_1 = I\left[\frac{K}{R_2} + \frac{\xi C_2}{R_3} + \frac{K}{R_3} + \frac{1}{R_2 R_3}\right]$$

$$\frac{V_1}{R_4} = \frac{I\left[\frac{K}{R_2} + \frac{\xi C_2}{R_3} + \frac{K}{R_3} + \frac{1}{R_2 R_3}\right]}{R_4 \quad \Delta Y}$$

3) a) hallar $V_C(s)$ aplicando Mallas

b) hallar V_C / $V(t) = 5V\,e^{-2t}\,u(t)$ con $R = 1K\Omega$, $C = 1\mu F$



$V(t) = 5\,e^{-2t}\,V\,u(t)$

$V(s) = 5\,\dfrac{1}{s+2}$

a) Mallas: $\mathbb{Z}\,\mathbb{I} = \mathbb{V}$

$$\begin{bmatrix} 4R & -2R \\ -2R & 3R + \frac{1}{sC} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V(s) \\ 0 \end{bmatrix} \quad ; \quad V_C(s) = I_2\,\frac{1}{sC}$$

$$I_2 = \frac{\text{Det } M I_2}{\text{Det } \mathbb{Z}} = \frac{\begin{vmatrix} 4R & V(s) \\ -2R & 0 \end{vmatrix}}{\begin{vmatrix} 4R & -2R \\ -2R & 3R + \frac{1}{sC} \end{vmatrix}} = \frac{2R\,V(s)}{12R^2 + 4\frac{R}{sC} - 4R^2} = \frac{2}{4}\,\frac{V(s)}{2R + \frac{1}{sC}}$$

$$I_2 = \frac{1}{2}\,\frac{sC}{sC\,2R + 1}\,V(s) \Rightarrow \frac{I_2}{sC} = V_C(s) = \frac{1}{4RC}\cdot\frac{1}{s + \frac{1}{2RC}}\cdot V(s)$$

$$\boxed{V_C(s) = \frac{V(s)}{4RC}\,\frac{1}{s + \frac{1}{2RC}}}\ \checkmark$$

I R

Verificación por Laplace y definición:

$$I = I_B + I_C = \frac{V_B}{2R} + sC\,V_C = \frac{V_C}{2R} + \frac{V_x}{2R} + sCV_C = \frac{V_C}{2R} + sCV_C + \frac{\cancel{R}}{2\cancel{R}}\,sCV_C$$

$$I = V_C\left(\frac{1}{2R} + sC + \frac{1}{2}sC\right)$$

$$V = V_A + V_B = V_A + V_x + V_C = 2IR + RsCV_C + V_C = \left(2R\left(\frac{1}{2R} + sC + \frac{1}{2}sC\right) + RsC + 1\right)V_C$$

$$V_C = V\,\frac{1}{2 + 4RsC} = \boxed{\frac{V}{4RC}\,\frac{1}{s + \frac{1}{2RC}}}\ \checkmark$$

$\dfrac{1-4RC}{2RC}$

b) $$V_C(s) = \frac{5}{s+2}\,\frac{1}{4RC}\,\frac{1}{s + \frac{1}{2RC}} = \frac{5}{4RC}\,\frac{1}{(s+2)(s + \frac{1}{2RC})} \Rightarrow \frac{A}{(s+2)} + \frac{B}{(s + \frac{1}{2RC})}$$

$A + B = 0$; $-A = +B$; $2B + \dfrac{A}{2RC} = \dfrac{5}{4RC} \rightarrow A\left(\dfrac{1}{2RC} - 2\right) = \dfrac{5}{4RC} \rightarrow A = \dfrac{2RC}{1-4RC}\cdot\dfrac{5}{4RC} = \dfrac{5}{2}\,\dfrac{1}{1-4RC}$

$$V_C(s) = \frac{5}{2}\,\frac{1}{1-4RC}\,\frac{1}{s+2} + \frac{5}{2}\,\frac{1}{4RC-1}\,\frac{1}{s + 2RC} = 2{,}51\left(\frac{1}{s+2} - \frac{1}{s + 2\times10^{-3}}\right)$$

$$\boxed{V_C(t) = 2{,}51\left(e^{-2t} - e^{-2\times10^{-3}t}\right)u(t)}$$

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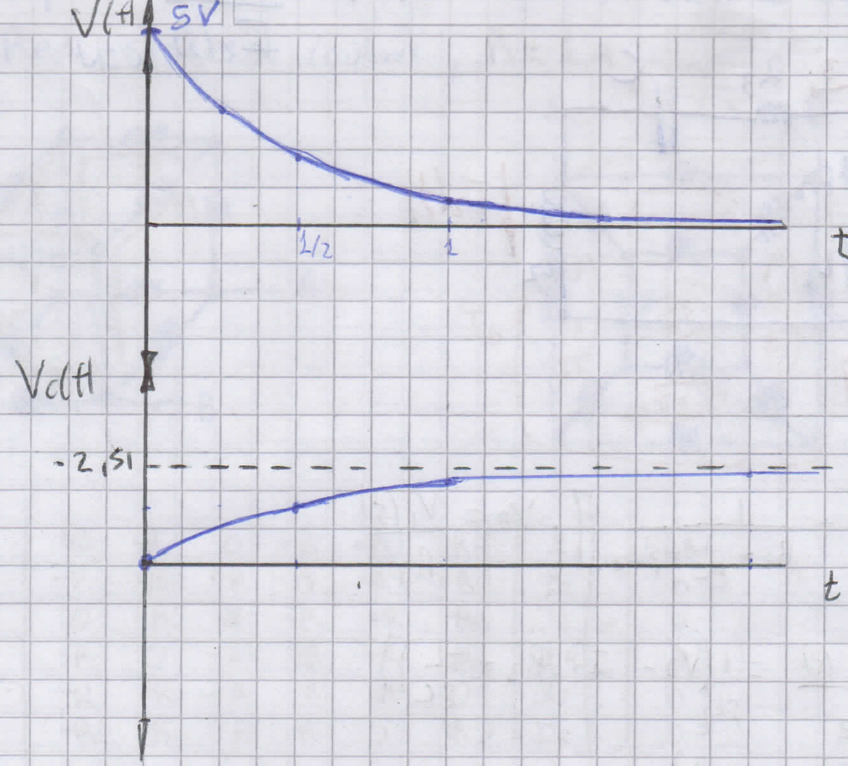
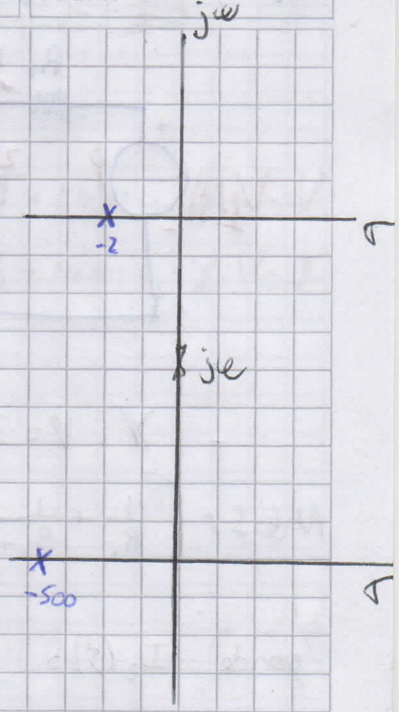


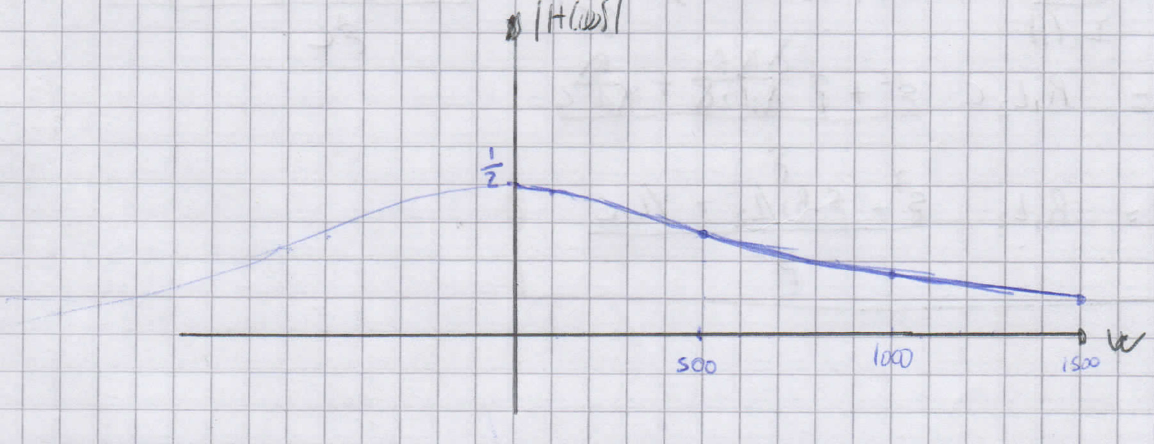


$$H(s) = \frac{V_c(s)}{V(s)} = \frac{1}{4RC} \quad \frac{1}{s + \frac{1}{2RC}} = 250 \quad \frac{1}{s + 500}$$

$$H(\omega) = H(s)\Big|_{s=j\omega} = 250 \quad \frac{1}{500 + j\omega} = \frac{250}{\sqrt{500^2 + \omega^2}} \, e^{-j\left[\operatorname{arctg}\left(\frac{\omega}{500}\right)\right]}$$

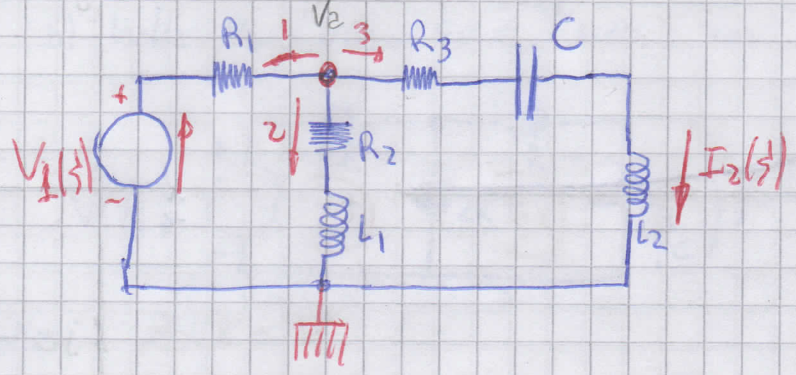
0,5 → 4

0,35



NOTA

4) hallar Impedancia de Transferencia $Z_T(s) = \left.\dfrac{V_1(s)}{I_2(s)}\right|_{CIN}$ por Nodos



$$\mathbb{Y} \cdot \mathbb{V} = \mathbb{I}$$

Nodo 1: $\left[\dfrac{1}{R_1} + \dfrac{1}{R_2 + sL_1} + \dfrac{1}{R_3 + \frac{1}{sC} + sL_2}\right] \cdot V_2 = \dfrac{V_1(s)}{R_1}$

donde $I_2(s) = \dfrac{V_{L_2}(s)}{sL_2} = \dfrac{1}{sL_2}\left(V_2 - I_2\left(R_3 + \dfrac{1}{sC}\right)\right)$

$$sL_2\left(I_2 + \frac{I_2}{sL_2}\left(R_3 + \frac{1}{sC}\right)\right) = \frac{V_2}{\cancel{sL_2}}$$

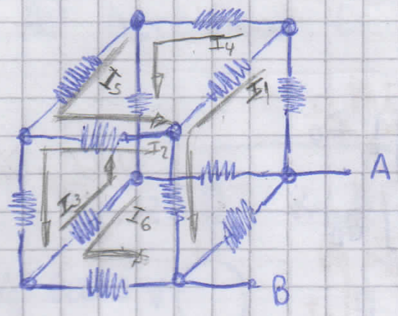
$$\frac{V_1}{R_1} = I_2\left(sL_2 + R_3 + \frac{1}{sC}\right) \rightarrow$$

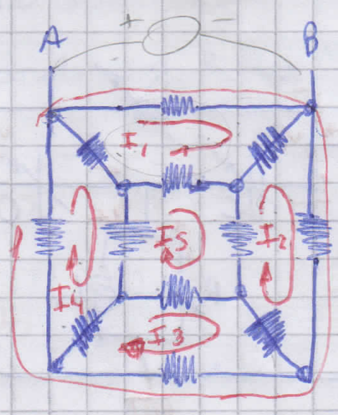
$$Z_T(s) = \frac{V_1(s)}{I_2(s)} = R_1 sL_2 + R_1R_3 + \frac{R_1}{sC} = \frac{s^2 R_1 L_2 C + sCR_1R_3 + R_1}{sC}$$

$$Z_T(s) = R_1 L_2 C \; \frac{s^2 + s\frac{CR_1R_3}{R_1L_2C} + \frac{R_1}{R_1L_2C}}{s}$$

$$\boxed{Z_T(s) = R_1 L_2 \; \frac{s^2 + s\,R_3/L_2 + 1/L_2C}{s}}$$

5) Red espacial (cubo) ; $R = 1\,\Omega$





6 → Mallas → $Z \cdot I = V$

8 → Nodos → $Y \cdot V = I$

Mallas

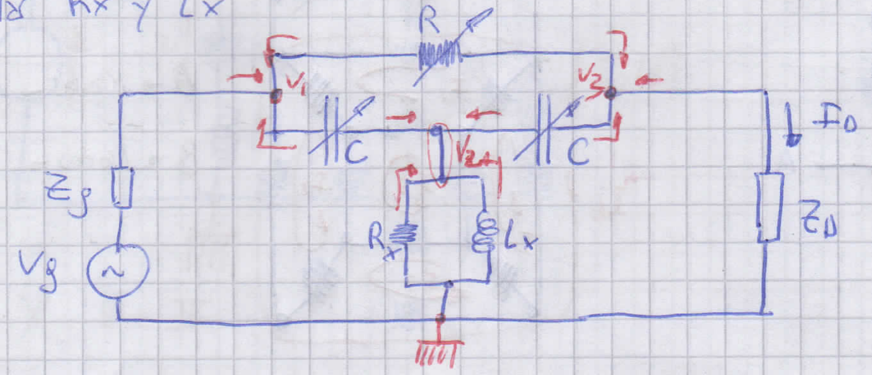
$$\begin{bmatrix} 4R & -R & 0 & -R & -R & -R \\ -R & 4R & -R & 0 & -R & -R \\ 0 & -R & 4R & -R & -R & -R \\ -R & 0 & -R & 4R & -R & -R \\ -R & -R & -R & -R & 4R & 0 \\ -R & -R & -R & -R & 0 & 4R \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} V_{AB} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Nodos:

6) $w = cte$

$C = C$ y con R Frec. Resonancia $\rightarrow I_D = 0$

hallar R_x y L_x



Nodos $\rightarrow$ Y. V = II

$$I_D(s) = 0 \quad ; \quad I_D(s) = \frac{V_3(s)}{Z_D(s)} = 0 \longrightarrow V_3(s) = 0$$

$$\begin{bmatrix} \frac{1}{Z_g} + \frac{1}{R} + sC & -sC & -1/R \\ -sC & sC + sC + \frac{1}{R_x} + \frac{1}{sL_x} & -sC \\ -1/R & -sC & \frac{1}{Z_D} + \frac{1}{R} + sC \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{V_g}{Z_g} \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta V_3 = \begin{vmatrix} \frac{1}{Z_g} + \frac{1}{R} + sC & -sC & V_g/Z_g \\ -sC & 2sC + \frac{1}{R_x} + \frac{1}{sL_x} & 0 \\ -1/R & -sC & 0 \end{vmatrix} = \frac{V_g}{Z_g}\left[s^2C^2 + \frac{2sC}{R} + \frac{1}{R_x R} + \frac{1}{sL_xR} \right] = 0$$

$$\Delta V_3 = 0 \rightarrow s^2C^2 + \frac{2sC}{R} + \frac{1}{R_x R} + \frac{1}{sL_xR} = 0 \quad ; \quad jw = s$$

$$-w^2C^2 + \frac{1}{R_xR} + j\left(\frac{2C}{R}w - \frac{1}{wL_xR}\right) = 0 + j0$$

$$\frac{1}{R_xR} - w^2C^2 = 0 \quad \wedge \quad \frac{2C}{R}w - \frac{1}{wL_xR} = 0$$

$$\boxed{R_x = \frac{1}{w^2C^2R}} \qquad L_x = \frac{1}{2\frac{C}{R}w^2R} \rightarrow \boxed{L_x = \frac{1}{2w^2C}}$$

NOTA

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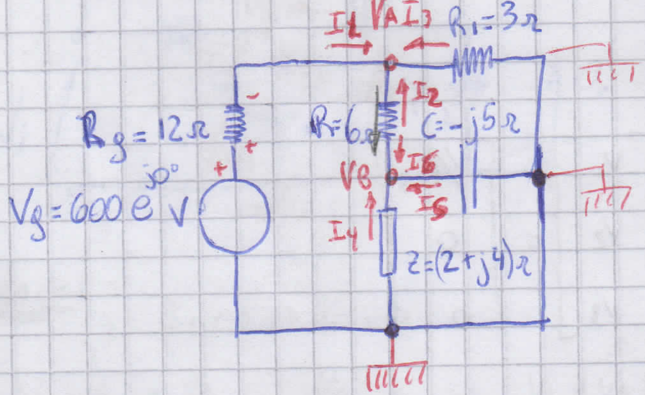
7) a) plantear ecuaciones de nodos

b) Determinar la I que circula por $R = 6\Omega$ y su caída de tensión



$$\begin{bmatrix} \frac{1}{R_g}+\frac{1}{R}+\frac{1}{R_1} & -\frac{1}{R} \\ -\frac{1}{R} & \frac{1}{R}+\frac{1}{C}+\frac{1}{Z} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} \frac{V_g}{R_g} \\ 0 \end{bmatrix}$$

$$Y \cdot V = I$$

NODO A) $I_1 + I_2 + I_3 = 0 \Rightarrow \dfrac{V_g - V_A}{R_g} + \dfrac{V_B - V_A}{R} - \dfrac{V_A}{R_1} = 0 \Rightarrow \dfrac{V_A}{R_g} - \dfrac{V_B}{R} + \dfrac{V_A}{R} + \dfrac{V_A}{R_1} = \dfrac{V_g}{R_g}$

NODO B) $I_4 + I_5 + I_6 = 0 \Rightarrow \dfrac{-V_B}{Z} - \dfrac{V_B}{C} + \dfrac{V_A - V_B}{R} = 0 \Rightarrow -\dfrac{V_A}{R} + \dfrac{V_B}{Z} + \dfrac{V_B}{C} + \dfrac{V_B}{R} = 0$

0,266 j

b) $I_R(s) = \dfrac{V_A - V_B}{R}$

$$V_A = \frac{\text{Det } MV_A}{\Delta Y} \rightarrow \text{Det } MV_A = \begin{vmatrix} V_g/R_g & -1/R \\ 0 & \frac{1}{R}+\frac{1}{C}+\frac{1}{Z} \end{vmatrix} = \frac{V_g}{R}\left[\frac{1}{R}+\frac{1}{C}+\frac{1}{Z}\right]$$

$$\Delta Y = \left(\frac{1}{R_g}+\frac{1}{R}+\frac{1}{R_1}\right)\left(\frac{1}{R}+\frac{1}{C}+\frac{1}{Z}\right) - \frac{1}{R^2} = \frac{1}{R_gR} + \frac{1}{R_gC} + \frac{1}{R_gZ} + \frac{1}{RC} + \frac{1}{RZ} + \frac{1}{R_1R} + \frac{1}{R_1C} + \frac{1}{R_1Z}$$

$$V_A = \frac{\frac{600e^{j0°}}{12\Omega}\left(\frac{1}{6\Omega} + j\frac{1}{5\Omega} + \frac{1}{2+j4}\right)}{\frac{1}{72} + j\frac{1}{60} + \frac{1}{24+j48} + j\frac{1}{30} + \frac{1}{12+j24} + \frac{1}{18} + j\frac{1}{15} + \frac{1}{6+j12}} = \frac{13,\overline{3}\, e^{j0}}{0,12\overline{7}\, e^{j0}} = 96,52\,V$$

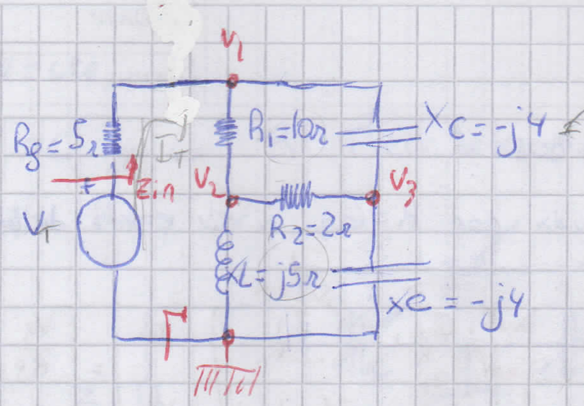
$$V_B = \frac{\text{Det}\begin{vmatrix} \frac{1}{R_g}+\frac{1}{R}+\frac{1}{R_1} & V_g/R_g \\ -1/R & 0 \end{vmatrix}}{\Delta Y} = \frac{\frac{600e^{j0}}{12\Omega}\left(\frac{1}{6\Omega}\right)}{\Delta Y} = \frac{8,\overline{3}\, e^{j0}}{0,12\overline{7}\, e^{j0}} = 65,2173\,V$$

$$\boxed{V_R = V_A - V_B = 96,52 - 65,22 = 31,3\,V}$$

$$\boxed{I_R = \frac{V_R}{R} = \frac{31,3\,V}{6\Omega} = 5,216\,A}$$

NOTA

8)



$Y = j\frac{1}{4}$

$$Z_{in} = \frac{1}{Y_{in}} = \frac{V_T}{I_T}$$

$Y = \frac{I}{V}$

$$\begin{bmatrix} \frac{3}{10} + j\frac{1}{4} & -1/10 & -j\frac{1}{4} \\ -1/10 & \frac{3}{5} - j\frac{1}{5} & -1/2 \\ -j\frac{1}{4} & -1/2 & \frac{1}{2} + j\frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V/R_g \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta Y = (-5\times10^{-3} + j0{,}0975) + (-5\times10^{-3} - j0{,}0175) + (0{,}0375 - j0{,}025) = 0{,}0275 + j0{,}055$$

$$\Delta Y = 0{,}0615\, e^{j63{,}43^\circ}$$

$0{,}2275$

$$\Delta Y \cdot V_1 = \det\begin{bmatrix} V/R_g & -1/10 & -j0{,}25 \\ 0 & 0{,}6 - j0{,}2 & -0{,}5 \\ 0 & -0{,}5 & 0{,}5 + j0{,}5 \end{bmatrix} = \frac{V}{R_g}\left[0{,}15 + j0{,}2\right]$$

$$V_T = V_{R_g} + V_1 = I \cdot R_g + I Z_1 = I(R_g + Z_1)$$

$$I = \frac{V_T}{R_g + Z_1} = \frac{V_T}{R_g} - \frac{V_1}{R_g} = \frac{V_1}{Z_1}$$

$$V_1 = \frac{\frac{V_T}{R_g}\left[0{,}15 + j0{,}2\right]}{0{,}0275 + j0{,}055} = \frac{V_T}{R_g} \cdot \bar{K}$$

$$I_T = \frac{1}{R_g}\left(V_T - V_T \frac{\bar{K}}{R_g}\right) = V_T\left(\frac{1}{R_g} - \frac{\bar{K}}{R_g^2}\right)$$

$$\frac{I_T}{V_T} = \frac{1}{R_g} - \frac{\bar{K}}{R_g^2}$$

$$Z_{in} = \frac{1}{\frac{1}{R_g} - \frac{\bar{K}}{R_g^2}} = \frac{1}{\frac{1}{5\Omega} - \frac{\frac{0{,}15 + j0{,}2}{0{,}0275 + j0{,}055}}{25\Omega}}$$

$$\boxed{Z_{in} = 16{,}35 - j\,11{,}89}$$

NOTA

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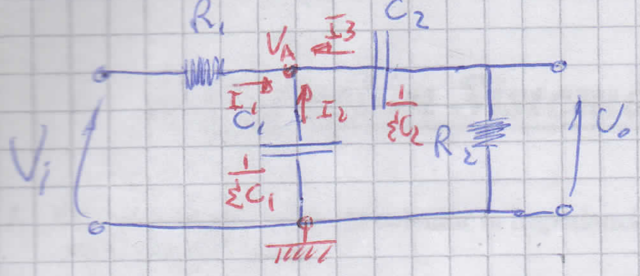
9) Hallar $H(s)$ por Nodos y Verificar por mallas



$$V_o = \frac{V_A}{\frac{1}{sC_2}+R_2} \cdot R_2 = V_A \frac{R_2}{\frac{1+sC_2R_2}{sC_2}}$$

$$V_o = V_A \cdot \frac{sC_2R_2}{1+sC_2R_2}$$

Nodos

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_i - V_A}{R_1} - \frac{V_A}{\frac{1}{sC_1}} - \frac{V_A}{\frac{1}{sC_2}+R_2} \Rightarrow \frac{V_i}{R_1} = V_A\left(\frac{1}{R_1} + sC_1 + \frac{1}{R_2+\frac{1}{sC_2}}\right)$$

$$V_i = V_A\left(1 + sC_1R_1 + \frac{sR_1C_2}{sC_2R_2+1}\right) = V_A\left(\frac{sC_2R_2+1+s^2C_1C_2R_1R_2+sC_1R_1+sR_1C_2}{sC_2R_2+1}\right)$$

$$V_i = V_o\left(\frac{\cancel{1+sC_2R_2}}{sC_2R_2}\right)\frac{s^2C_1C_2R_1R_2+s(C_2R_2+C_1R_1+R_1C_2)+1}{\cancel{sC_2R_2+1}}$$

$$\frac{V_o}{V_i} = H(s) = \frac{sC_2R_2}{s^2C_1C_2R_1R_2+s(C_2R_2+C_1R_1+R_1C_2)+1} = \frac{C_2R_2}{C_1C_2R_1R_2} \cdot \frac{s}{s^2+s\left(\frac{1}{C_1R_1}+\frac{1}{C_2R_2}+\frac{1}{R_2C_1}\right)+\frac{1}{R_1R_2C_1C_2}}$$

$$\boxed{H(s) = \frac{\frac{1}{R_1C_1}\, s}{s^2+s\left(\frac{1}{C_1R_1}+\frac{1}{C_2R_2}+\frac{1}{C_1R_2}\right)+\frac{1}{R_1R_2C_1C_2}}}$$

NOTA